Context-dependent substitutability: impacts on environmental preferences and discounting

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Abstract

Willingness-to-pay for environmental quality as well as discounting are two crucial concepts in environmental economics. Both are linked to the substitutability between consumption and environmental quality. In classical economic models, two goods are exogenously defined as complements or as substitutes. In this paper, we develop a theoretical framework in which goods can be either complements or substitutes depending on the context, i.e., on their quantities. We analyze the impacts of a context-dependent substitutability on (i) environmental preferences and (ii) the rates at which consumption and environmental quality are discounted. We show that, in a context of substitutability, a decision-maker’s WTP can decrease as her income grows and increase as environmental quality grows. We also show that the sign of the so-called substitution effect in discounting can change whether the goods are substitutable or not. This impacts the values of the economic and environmental discount rates. It is particularly well-illustrated by ecological or economic shocks, which may induce a sudden change in the decision-maker’s behavior toward consumption and the environment.

Key words: substitutability, complementarity, contextual preferences, willingness-to-pay for environmental quality, discounting

JEL Code: Q50, D90, D91
1 Introduction

Are consumption of man-made goods\footnote{Throughout the paper, we abusively use the words “income” and “consumption” to refer to the “economic outcome” of a prospect. A reader worried about the rigor of this usage can think of Hicksian income as a reference. The empirical literature also refers to available, or residual income to signify this quantity.} and environmental quality complements or substitutes? This question is fundamental to evaluating projects with economic and environmental intertemporal impacts, for two reasons. Both the concepts of willingness-to-pay (WTP) for the environment and of dual-rate discounting are closely linked to the concept of substitutability. This means that the way the environment is valued at a given time and the way we should discount this value over time depend on substitutability. This is crucial to address environmental issues such as climate change or biodiversity conservation, in which discounting is a central question (Yang, 2003, Guesnerie, 2004, Tol, 2004, Weikard & Zhu, 2005, Hoel & Sterner, 2007, Sterner & Persson, 2008, Kögel, 2009, Gollier, 2010, Traeger, 2011, Echazu \textit{et al.}, 2012).

In economic models which consider a two-arguments utility function,\footnote{Consumption of man-made goods and environmental quality} and in particular in models addressing the ecological discounting issue, substitutability is fixed by exogenous parameters. When they are not considered as independent (Heal, 1998), the two goods are either considered as substitutes, even perfect substitutes in some models (Nordhaus, 2008, Sterner & Persson, 2008, Neumayer, 2013) or as complements (Weikard & Zhu, 2005). Either of the two assumptions determines the sign of the substitution effect in the discount rates. As a consequence, the sign of this effect never depends on the wealth of the economy or on its environmental quality.

Empirical results suggest, however, that people substitute consumption of produced goods and environmental quality in a different way according to their income\footnote{Shogren \textit{et al.} (1994) experimentally showed the existence of a relation between the impact of income on WTP and the substitutability between a market good and a nonmarket good.}. Indeed, both empirical literature on environmental preferences and on time preferences points out such an income effect. WTP for improving environmental quality varies with income. So does the rate at which people discount the future. Income effects on WTP are mainly found positive and, more accurately, the income elasticity of WTP is often found between 0 and 1 (Kristrom & Riera, 1996, Flores & Carson, 1997, Jacobsen & Hanley, 2009). Some studies, however, report negative income effects (McFadden & Leonard, 1993, McFadden, 1994, Horowitz, 2002, Huhtala, 2010), which means that in some contexts, WTP for the environment decreases as income rises. Income effects on the discount rate are mostly reported negative (Hausman, 1979, Lawrance, 1991, Harrison \textit{et al.}, 2002, Tanaka \textit{et al.}, 2010). At a macroeconomic level, developing and developed countries also exhibit different patterns in terms of (stated) preference for the environment and discount rate.

The usual interpretation of such empirical evidence is that low-income people and high-income people have “different preferences,” either in terms of environmental preferences or in terms of time preferences. Does it mean that goods are considered as...
substitutes by a part of a population and complements by another? In that case, the preferences of the two groups would be fundamentally different, and the utility models of the two behaviors would require significantly different parameters. One may argue, however, that the difference in revealed / stated preferences is contingent on the income or situation of the individuals, and is not driven by fundamentally different preferences. Would an individual from the “low-income group” behave like a poor if given the endowments of a rich? At countries level, won’t preferences of developing country change once developed? In this case, it would be helpful to have a model that represents both the stated preferences associated with complementarity between goods and that associated with substitutability, depending on the context of the choice. This could provide a theoretical explanation on why low-income and high-income people have different stated WTP and different revealed discount rates. Unfortunately, no single theoretical framework allows for both substitutability and complementarity without changing exogenous parameters in preference representation.

In this paper, we propose a theoretical framework in which the substitutability between economic and environmental goods is not exogenously imposed but depends on the context. The context is defined by the current level of income and of environmental quality. The three kinds of relationship (independence, substitutability, complementarity) are allowed and can vary according to income and environmental quality. Roughly speaking, if applied to high-income and low-income people, all individuals would behave according to the same preferences but the expression of these preferences would differ depending on the context, so that both complementarity and substitutability between goods are possible in different contexts. Providing an empirical support for our theory is beyond the scope of the present work. We suggest, however, some empirical testing for further research.

We analyze the impacts of a context-dependent substitutability on WTP changes with income and environmental quality. We show that, within our theoretical framework, positive income effects are possible both when the goods are substitutes and complements, but negative income effects are only possible in a context of substitutability between goods. We also show that an improvement of environmental quality will have a negative effect on WTP for the environment in general, but that it may induce a positive effect on WTP in a context of substitutability between goods. Besides we examine the impacts of a context-dependent substitutability on discounting and especially on the approach of dual-rate discounting. Our framework does not change the way goods are discounted but the values of the discount rates. An important result is that the discount rate does not only depend on the prospect for (economic and environmental) growth but also on the current level of economic and environmental outcomes. Our contribution to the literature on dual-rate discounting is to show that the substitution effect has opposite impacts on the two discount rates (economic and environmental) whether goods are substitutes or complements. The interest of the contribution is well-illustrated when an ecological (or economic) shock occurs. In such a case, the decision-maker can change her perception of the two goods, for example by considering initially the two goods as complements and
as substitutes once the shock has occurred.

We illustrate our results on environmental preferences and discounting through a specific utility function that satisfies our assumptions. This specific function gives a support to detail what is meant by context. In this example, context refers to economic-ecological poverty (domain of substitutability between goods) and economic-ecological wealth (domain of complementarity between goods). We define the concept of economic-ecological wealth, by not only considering people’s consumption (or income) levels but a combination of their consumption level and environmental quality. Therefore, people are said to be economic-ecologically poor if they consider income and environmental quality as substitutes given their current endowment. In this case, they are more likely to sacrifice environmental quality for more consumption. Whereas people are said to be economic-ecologically rich if they consider the two goods as complements given their current endowment. In this case, they are more sensitive to the environment and feel a distaste effect\(^4\) when experiencing an environmental quality loss (i.e., they might consume less produced goods).

The paper is organized as follows. Section 2 introduces the different concepts of substitutability and puts forward the link which exists (i) between willingness-to-pay and substitutability and (ii) between discounting and substitutability. Section 3 presents our general theoretical framework and introduces a particular utility function that matches our assumptions. Section 4 derives the impacts of a context-dependent substitutability on environmental preferences. Section 5 examines the impacts of a context-dependent substitutability on discounting. Section 6 suggests some empirical testing for the theory we develop. In Section 7, we conclude on the practical implications of our results and suggest future research directions.

2 Three closely linked concepts: substitutability, willingness-to-pay and discounting

In this section, we introduce the concept of substitutability and we show formally (i) the link between substitutability and the willingness-to-pay for the environment and (ii) the link between substitutability and discounting.

2.1 Substitutability between goods: different concepts

Preferences over two goods can be observed through the trade-offs decision-makers reveal while “consuming” the two goods. Consider two “goods” contributing to the well-being

\(^4\)So-called by Michel & Rotillon (1995).
of a consumer or of a society: an aggregated conventional man-made good, denoted by \( x \), and an aggregated environmental good, denoted by \( y \). We are interested in the way the substitutability between the two goods affects the decision-maker’s choices between the two goods and over time.

We assume that increasing the consumption of each good increases well-being, but at a decreasing rate. Preferences over these two goods are represented by a utility function which is increasing and concave in each of its arguments.

**Assumption 1** The utility function \( u(\cdot, \cdot): \mathbb{R}^{+2} \rightarrow \mathbb{R} \) is continuously and twice differentiable, strictly increasing and concave in each of its arguments.

\( \mathbb{R}^{+2} \) is defined as the bi-dimensional set of real strict positive numbers, with \( \mathbb{R}^{+} = [0, +\infty[ \).

Further, we use the following notations for the derivatives of the utility function:

\[
\begin{align*}
    u_i &\equiv \frac{\partial u}{\partial i}, \\
    u_{ii} &\equiv \frac{\partial^2 u}{\partial i^2}, \\
    u_{ij} &\equiv \frac{\partial^2 u}{\partial i \partial j}, \\
    \dot{u} &\equiv \frac{du}{dt},
\end{align*}
\]

where \( i \neq j \) are either the variable \( x \) or \( y \) and \( t \) represents time.

There are different ways of defining substitutability. Samuelson (1974) gives an exhaustive overview of these various definitions. Substitutability is generally defined either in the Edgeworth-Pareto (E-P) sense (Edgeworth, 1925, Pareto, 1909) or in the Hicks sense (Hicks, 1932). The E-P definition relies on the sign of the cross derivative of the utility function:

\[
\begin{align*}
    u_{xy} < 0 \quad & x \text{ and } y \text{ are substitutes or rivals} \\
    u_{xy} > 0 \quad & x \text{ and } y \text{ are complements} \\
    u_{xy} = 0 \quad & x \text{ and } y \text{ are independents}
\end{align*}
\]

The Hicks definition, which is used to characterize substitutability in the case of homothetic preferences only,\(^7\) is based on the so-called “1932 elasticity of substitution”\(^8\)

\[
\chi_{xy} \equiv \frac{u_x u_y}{u_{xy}}
\]

If \( \chi_{xy} \rightarrow 0 \) then \( x \) and \( y \) are perfect complements. If \( \chi_{xy} \rightarrow \infty \) then \( x \) and \( y \) are perfect substitutes. If \( \chi_{xy} = 1 \) then preferences over \( x \) and \( y \) are represented by a Cobb-Douglas function.

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5. This could either refer to “consumption” or “economic production.” In a meta-analysis examining if there is an income effect on the willingness-to-pay for biodiversity conservation, Jacobsen & Hanley (2009) state that “GDP per capita seemed to perform as well as an explanatory variable as respondent’s mean stated income, indicating that it is wealth in society as a whole which determines variations in WTP. Even if large variation, our main conclusion is that the demand for biodiversity conservation rises with a nation’s wealth, but the income elasticity of willingness to pay is less than one.”

6. This condition is less restrictive than strict concavity.

7. Preferences are homothetic if and only if they are of the form \( \Phi(x, y) = F(f(x, y)) \) where \( F \) is a monotone increasing function, \( f \) is a homogeneous function of degree one, that is, \( f(kx, ky) = kf(x, y) \) for any \( k > 0 \).

8. That is \( \frac{d \ln(x)}{d \ln(y)} = \frac{u_x u_y}{u_{xy}} \left( x u_x + y u_y \right) \), which is equal to \( u_x u_y / u_{xy} \) when \( u \) is homogeneous of degree one, implying \( u = xu_x + yu_y \).
utility function. And a CES utility function implies $\chi_{xy} = \sigma$ where $\sigma$ is the elasticity of substitution between $x$ and $y$.\(^9\)

To avoid restricting our analysis to homothetic preferences, we use the Edeworth-Pareto (E-P) definition of substitutability.

The E-P criterion characterizes the substitutability between the two goods but does not differentiate the effect of consumption $x$ on satisfaction derived from environmental quality $y$ from the effect of environmental quality on consumption satisfaction.\(^10\) As Neumayer highlights, if we consider two products A and B, then

"for some production purposes A and B are almost near-perfect substitutes with almost linear isoquants. But for other purposes, A has some desirable properties that B does not have. Hence, A can substitute for the totality of B, but not vice-versa." (Neumayer, 2013, p. 67)

Considering substitutability in utility, this reasoning can be related to Lancaster’s theory of consumption: “The good, per se, does not give utility to the consumer; it possesses characteristics, and these characteristics give rise to utility. In general, a good will possess more than one characteristic, and many characteristics will be shared by more than one good” (Lancaster, 1966). For instance, $x$ could be a perfect substitute for $y$ because it possesses all the characteristics of $y$, but $y$ might not replace totally $x$ because it lacks some of the characteristics of $x$. Therefore, in order to measure the degree of substitutability between the two goods, we use the concepts of cross elasticities of marginal utility.

**Definition 1** The elasticity of marginal utility for consumption with respect to environmental quality, denoted by $\eta_{x,y}$, is the variation of marginal utility for consumption resulting from a change of the environmental quality:

$$\eta_{x,y} \equiv -\frac{\partial u_x}{\partial y} = -y \frac{u_{xy}}{u_x}. \quad (2)$$

The elasticity $\eta_{x,y}$ measures the impact of a change in environmental quality on the satisfaction derived from consumption.

**Definition 2** The elasticity of marginal utility for environmental quality with respect to consumption, denoted by $\eta_{y,x}$, is the variation of marginal utility for environmental quality resulting from a change in consumption:

$$\eta_{y,x} \equiv -\frac{\partial u_y}{\partial x} = -x \frac{u_{yx}}{u_y}. \quad (3)$$

\(^9\)The CES utility function is such that $u^{CES}(x,y) = (bx^{1-\frac{1}{\sigma}} + (1-b)y^{1-\frac{1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$ with $\sigma$ the elasticity of substitution between $x$ and $y$ and $b$ the value share for consumption.

\(^{10}\)As $u_{xy} = u_{yx}$ (Schwarz’ theorem).
The elasticity $\eta_{y,x}$ measures the impact of a change in consumption on the satisfaction derived from environmental quality.

Remark: Both elasticities $\eta_{x,y}$ and $\eta_{y,x}$ have the same sign (which is opposite to the sign of $u_{xy}$). This means that if $x$ is a substitute (resp. complement) for $y$, then $y$ is also a substitute (resp. complement) for $x$. The degree of substitutability between $x$ and $y$ is not the same as the degree of substitutability between $y$ and $x$, however, since $\eta_{x,y} \neq \eta_{y,x}$.

### 2.2 Substitutability and willingness-to-pay

The willingness-to-pay is usually defined as the maximum amount a person would be willing to pay in order to receive something desirable (e.g., increasing environmental quality) or to avoid something undesirable (e.g., pollution). Formally, with the same notations as before, we have

$$u(x - WTP, y + \Delta y) = u(x, y). \quad (4)$$

Thus, this person whose preferences are represented by the utility function $u$ is indifferent between (i) giving the amount $WTP (= \Delta x > 0)$ for improving environmental quality of $\Delta y$ and (ii) not paying for environmental improvements, thus staying with the initial environmental quality $y$.

At the margin, WTP for a change of environmental quality is equal to the marginal rate of substitution between consumption and environmental quality:

$$WTP = \frac{u_y}{u_x}. \quad (5)$$

The variation of WTP with income is

$$\frac{\partial WTP}{\partial x} = \frac{u_{xy}u_x - u_{xx}u_y}{u_x^2}. \quad (6)$$

Therefore, this change of WTP with income depends on the substitutability between goods (characterized by $u_{xy}$). In the empirical literature, reference is often made to the income elasticity of WTP, which is formally defined as:

$$\varepsilon_{WTP}^x = \frac{\partial WTP}{\partial x} = \frac{\partial WTP}{WTP} \cdot \frac{x}{WTP}. \quad (7)$$

Mostly, it is found that $0 < \varepsilon_{WTP}^x < 1$, which means that WTP increases less than proportional to income. The case of a negative income elasticity, i.e., $\varepsilon_{WTP}^x < 0$, can also occur, even if it is underrepresented in empirical results. As $x > 0$ and $WTP > 0$, the sign of $\varepsilon_{WTP}^x$ is of the same sign as $\frac{\partial WTP}{\partial x}$, hence the income elasticity of WTP depends

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[11] If the consumer is indifferent between two situations, then the variation of her utility is nil:

$$du = u_x dx + u_y dy = 0 \Leftrightarrow -\frac{du}{dy} = \frac{u_x}{u_y} = TMSx,y.$$ The marginal rate of substitution is indeed often defined as the marginal willingness-to-pay for consuming an additional marginal quantity of a good.
on substitutability.

Although empirical literature focuses on income effects on WTP, which is well-illustrated in the literature on benefit transfers for instance, a similar thought should be given to the variations of WTP with environmental quality. We have
\[
\frac{\partial WTP}{\partial y} = \frac{u_{yx}u_y - u_{xy}u_y}{u_y^2}.
\]
(8)
This expression depends on substitutability too. And the environmental elasticity of WTP is such that
\[
\varepsilon_y^{WTP} = \frac{\partial WTP}{WTP \frac{\partial y}{y}} = \frac{\partial WTP}{\partial y} \frac{y}{WTP}.
\]
(9)
As income or environmental quality improves, how does WTP change? In Eqs. (6) and (8), the sign of the income and the environmental effect on WTP depends on the cross derivative of \(u\), hence on the substitutability between consumption \(x\) and environmental quality \(y\). If \(u_{xy} > 0\), which means that goods are complements, then \(\varepsilon_x^{WTP}\) is positive and \(\varepsilon_y^{WTP}\) is negative. But this is not necessarily the case when goods are substitutable, i.e., if \(u_{xy} < 0\). In that case, the WTP can decrease as income rises and it can increase as environmental quality rises. This contradicts the standard intuition that WTP should increase when income increases and decrease when environmental quality increases. In Section 4, we derive the impacts of this dependence on environmental preferences and clarify in which conditions the different empirical results described in Section 1 can be found.

2.3 Substitutability and discounting


Dual-rate discounting is an alternative to considering the evolution of the relative price of the environmental good over time and discounting it with the usual economic (or consumption) discount rate. In a cost-benefit analysis, a project that yields economic and environmental outcomes can be evaluated either by monetizing the environmental outcome at each date and applying the economic discount rate, or by discounting the environmental outcome with an appropriate environmental discount rate and monetizing the net environmental value with the present price of the environment. There are thus two equivalent ways to determining the net present value (NPV) of the project. To ease the interpretation, let us describe these two alternatives in a discrete time framework, before adopting the continuous time framework used in the literature.


\(^{13}\)As first suggested by Fisher & Krutilla (1975).
Consider a project starting at time $t = 0$ with time horizon $T$, and resulting in a sequence of outcomes $\{(x_t, y_t)\}_{t=0,1,\ldots,T}$. The rates at which next period’s consumption and environmental quality are discounted at time $t - 1$ are denoted respectively by $r_x^t$ and $r_y^t$ (with the convention $r_x^0 = r_y^0 = 0$). Considering $x_t$ as the numéraire good, $p_0$ is the accounting price for the environment at initial time $t = 0$. The relative price of the environmental good at time $t$, that is, the marginal willingness-to-pay for the environment at time $t$, is denoted by $p_t$. The NPV of the project is then

$$NPV \left( \{(x_t, y_t)\}_{t=0,1,\ldots,T} \right) = \sum_{t=0}^{T} \frac{x_t + p_t y_t}{\Pi_{s=0}^{t}(1 + r_x^s)}$$

where $\delta$ is the rate of pure preference for the present.\(^{15}\)

Along any trajectory, we can derive the discount rates relative to consumption and to environmental quality.\(^{16}\) The consumption (resp. environmental) discount rate represents the willingness to trade off present for future consumption (resp. environmental quality). By definition,\(^{17}\) the consumption discount rate, $r_x$, is the rate at which the value of a small increment of consumption changes as time changes.\(^{18}\) From this definition, using the elasticity $\eta_{x,y}$ defined in Eq. (2), we obtain the consumption discount rate\(^{19}\)

$$r_x = \delta + g_x \eta_{x,x} + g_y \eta_{x,y} ,$$

where $g_x = \frac{\dot{x}}{x}$ and $g_y = \frac{\dot{y}}{y}$ are the growth rates of consumption and environmental quality.

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\(^{14}\)The accounting price is the value of a unit of environmental quality $y$ expressed in units of consumption $x$. It corresponds to the marginal willingness-to-pay for improving environmental quality and is given by the marginal rate of substitution between $x$ and $y$.

\(^{15}\)Also called the utility discount rate, this is the rate at which the weight of the bi-attributes utility declines over time.

\(^{16}\)Usually, the expression of the discount rate is derived from the Keynes-Ramsey rule, along an optimal path. The derivation we use in Appendix A is valid along any given trajectory. We thus do not reduce our analysis to optimal trajectories.

\(^{17}\)See Heal (1998, p.77).

\(^{18}\)We can also say that $r_x$ is the rate of change of marginal utility for consumption as time changes. $r_x = \frac{\dot{U}_x}{U_x}$ (resp. $r_y = \frac{\dot{U}_y}{U_y}$) where $U(x, y, t) = u(x_t, y_t)e^{-\delta t}$.

\(^{19}\)See Appendix A.
Such a discount rate can also be defined for environmental quality, using the elasticity \( \eta_{y,x} \) defined in eq. (3), providing the environmental discount rate

\[
20 \quad r^y = \delta + g_y \eta_{y,y} + g_x \eta_{y,x} .
\]  

Both discount rates are the sum of three terms. The first term is the rate of pure preference for the present. The second term corresponds to the so-called growth effect. It depends on the elasticity of marginal utility for consumption \( \eta_{x,x} \) for the economic discount rate \( r^x \), and on that for environmental quality \( \eta_{y,y} \), for the ecological discount rate \( r^y \). The third term is the so-called substitution effect. It depends on the cross elasticities of marginal utility introduced in Definitions 1 and 2. The two discount rates depend on the substitutability between the two goods through \( \eta_{x,y} \) for \( r^x \) and \( \eta_{y,x} \) for \( r^y \).

2.4 Substitutability and preference representation

The two previous subsections exhibited that both the WTP for the environment and the (economic and environmental) discount rates depend on substitutability between the economic good and the environment.

In the literature relative to discounting, an assumption is made about this substitutability. Either i) the goods are assumed to be independent and the substitution effect is neglected, or ii) the goods are assumed to be substitutes, and even perfect substitutes as it is the case in Nordhaus’ model\(^{21}\) (Nordhaus, 1991, 1994, Nordhaus & Boyer, 2000, Nordhaus, 2008), or iii) they are supposed to be complements (Weikard & Zhu, 2005). Generally, substitutability is indirectly determined by exogenous parameters (the example below describes these parameters). This is the case for most contributions on dual-rate discounting, which use either an isoelastic CES utility function (Guesnerie (2004), Hoel & Sterner (2007), Gollier (2010), Kögel (2009), Traeger (2011)), an isoelastic Cobb-Douglas utility function (Gollier, 2010), or an isolastic VES utility function\(^{22}\) (Echazu et al., 2012). All are functions representing homothetic preferences.

Let’s analyze the example of an isoelastic CES utility function:

\[
20 \quad u^{isoCES}(x, y) = \frac{1}{1 - \mu} \left( bx^{1-\nu} + (1 - b)y^{1-\nu} \right)^{\frac{1-\mu}{1-\nu}}
\]  

where \( b \) is the weight given to consumption, \( \mu \) is the resistance to intertemporal substitution and \( \nu \) is the resistance to substitution between goods. The cross derivative of \( u^{isoCES} \) is

\[
20 \quad u^{isoCES}_{xy} = b(1 - b)(\nu - \mu)x^{-\nu}y^{-\nu}(bx^{1-\nu} + (1 - b)y^{1-\nu})^{\frac{1-\mu}{1-\nu}-2} .
\]  

This cross derivative is positive if \( \nu > \mu \) and negative if \( \nu < \mu \). Therefore, substitutability between consumption and environmental quality depends only on the sign of the difference between \( \nu \) and \( \mu \), two exogenous parameters characterizing the utility function. In

\(^{20}\)The expression (13) for \( r^y \) was first derived by Weikard & Zhu (2005).

\(^{21}\)It has been highlighted by Sterner & Persson (2008), Neumayer (2013).

\(^{22}\)Variable Elasticity of Substitution function. \( u^{VES}(x, y) = \left[ by^{1-a} + (1 - b)(y^{a}x^{1-a})^{1-a} \right]^{\frac{1}{1-a}} \) where \( 0 \leq a < 1 \), \( b \) is the value share of environmental quality.
particular, the kind of relationship between the two goods does not depend on their quantities.

One may argue, however, that a decision-maker may judge two goods as substitutes in a context and complements in another (depending on her needs and current condition). Referring again to Lancaster’s concept of consumer preferences, marginal utility is derived from changes in the characteristics of the bundle of “consumed” goods. The two goods may be complements for some characteristics and substitutes for others. In two different contexts (e.g., if endowments are different), the characteristics of the two goods may be valued differently, or, more precisely, the decision-maker may attribute value to different characteristics. For example, someone rich (high-income) may judge that environmental quality and private income are complements in increasing utility, as the characteristic of interest for her may be related to expensive recreational activities requiring both high income and high environmental quality. On the contrary, someone poor (low-income) may judge private consumption and environmental quality as substitutes because the two goods contribute redundantly to a same characteristic, such as housing or heating, related to the satisfaction of basic needs. Faced with a set of similar choices, these two individuals may exhibit different stated preferences, reflecting complementarity for one and substitutability for the other.

In the standard modeling framework, such a difference in revealed preferences is possible only if the two decision-makers preferences are fundamentally different in the sense that one deems the two goods as substitutes while the other considers them as complements. To represent these preferences, one must assume that the parameters in the utility function are different. Would the poor become rich, will she still value the environment and the other consumption goods as substitutes? It is more likely that she exhibits the same behavior as the rich does. By relying on an absolute characterization of substitutability between goods, the standard representation of preferences does not allow for such cases. Our view is that such a difference in stated preferences should not be supported by fundamentally different preferences but by the same preferences resulting in different statements depending on the context. Roughly speaking, the two decision-makers should have the same preferences, allowing the goods to be judged as complements in some contexts and as substitutes in others. As a consequence, in our preferences setting, substitutability is not an assumption anymore but depends on the context. This has important consequences on the way the willingness-to-pay for the environment evolves as the income and environmental quality evolve, and on the way the environment may be discounted in different contexts, for example by countries with different levels of development.

3 General theoretical framework

In this section, we develop a theoretical framework which allows substitutability to depend on the context.
3.1 Context-dependent preferences

We start by specifying that the context is defined by the quantity of the two goods providing utility.

**Definition 3 (Context)** The context is defined as the current quantity of the two arguments of the utility function, i.e., \((x, y) \in \mathbb{R}^{+2}\).

We assume that there is a function specifying if the two goods are substitutable or not, given the context.

**Assumption 2 (Context-dependent substitutability)** There is a continuous function \(\Theta : \mathbb{R}^{+2} \rightarrow \mathbb{R}\) such that \(x\) and \(y\) are complements if \(\Theta(x, y) > 0\), substitutes if \(\Theta(x, y) < 0\), and independent if \(\Theta(x, y) = 0\).

The meaning of this assumption will be clear once we specify an example.

Note that the last domain corresponds to the frontier between the substitutability domain and the complementarity domain. \(\Theta\) can be either an increasing or decreasing function of \(x\) and \(y\).\(^{23}\) An example of such a function is specified in Subsection 3.2. Assumption 2 is fundamental in that it can change the results on environmental preferences and discounting described in the previous section. This is what we analyze in Sections 4 and 5. Then the rest of the analysis has to rely on a utility function satisfying the following properties (Assumptions 1 and 2):

\[
\forall x, y \in \mathbb{R}^{*} \begin{cases} 
u_x > 0 \\
u_y > 0 \\
u_{xx} < 0 \\
u_{yy} < 0 \\
u_{xy} > 0 \forall (x, y) : \Theta(x, y) > 0 \\
u_{xy} < 0 \forall (x, y) : \Theta(x, y) < 0 \\
u_{xy} = 0 \forall (x, y) : \Theta(x, y) = 0
\end{cases}
\]

For the next sections, we specify the signs of the elasticities and the cross elasticities of marginal utility:

\[
\eta_{x,x} = -x \frac{u_{xx}}{u_x} > 0 . \quad (16)
\]

\[
\eta_{y,y} = -y \frac{u_{yy}}{u_y} > 0 . \quad (17)
\]

As \(u_x > 0, u_{xx} < 0\) and \(u_y > 0, u_{yy} < 0 \forall x, y > 0\), the elasticities of marginal utility for consumption \(\eta_{x,x}\) and for environmental quality \(\eta_{y,y}\) are positive everywhere.

\[
\eta_{x,y} = -y \frac{u_{xy}}{u_x} \leq 0 . \quad (18)
\]

\(^{23}\) A trivial candidate to characterize the substitutability of a context is the cross derivative of the utility function. We shall see, however, that other functions (with the same sign on the domain of definition of \(u(\cdot, \cdot)\)) can be used and provide interesting interpretations.
\[ \eta_{y,x} = -x \frac{u_{xy}}{u_y} \leq 0. \]  
(19)

\( \eta_{x,y} \) and \( \eta_{y,x} \) are positive if and only if \( u_{xy} < 0 \). They are negative if and only if \( u_{xy} > 0 \) and equal to zero if and only if \( u_{xy} = 0 \).

### 3.2 An example of utility function with context-dependent substitutability

To illustrate our theoretical framework, we use the following example in which the context is associated with “economic-ecological wealth” (defined below).

For any \((x, y) \in \mathbb{R}^2_+\), consider the function
\[
 u(x, y) \equiv x^\gamma y^\omega - \frac{\theta}{x^\alpha y^\beta} = x^\gamma y^\omega \left(1 - \frac{\theta}{x^{\gamma + \alpha} y^{\omega + \beta}}\right),
\]  
(20)

where \( \theta > 0 \) is a given parameter and \( 0 < \gamma, \alpha, \omega, \beta < 1 \). This function satisfies the conditions of Assumptions 1 and 2.\(^{24}\) For simplicity, we call it the context-dependent substitutability (CDS) function.\(^{25}\)

The function is the sum of two terms. The first term is positive and increasing with \( x \) and \( y \), and the second term is negative and also increasing in \( x \) and \( y \). The higher \( x \) and \( y \), the more negligible the second term. To ease the interpretations, we shall refer to this second term as the “needs” part of the utility. In the limiting case where \( \theta = 0 \), this “needs” term vanishes and the function reduces to the classic Cobb-Douglas utility function case (namely the “classic” term of the utility function). On the contrary, when \( x \) and \( y \) are low, the second term of the sum is not negligible and reduces utility.

The cross derivative of the CDS utility function is:\(^{26}\)
\[
 u_{xy} = u_{yx} = \gamma \omega x^{\gamma - 1} y^{\omega - 1} - \theta \alpha \beta x^{-\alpha - 1} y^{-\beta - 1}.
\]  
(21)

Its sign depends on the context as follows:
\[
\left\{ \begin{array}{ll}
 u_{xy} > 0 & \forall (x, y) : x^{\gamma + \alpha} y^{\omega + \beta} > \frac{\theta \alpha \beta}{\gamma \omega} \\
 u_{xy} < 0 & \forall (x, y) : x^{\gamma + \alpha} y^{\omega + \beta} < \frac{\theta \alpha \beta}{\gamma \omega} \\
 u_{xy} = 0 & \forall (x, y) : x^{\gamma + \alpha} y^{\omega + \beta} = \frac{\theta \alpha \beta}{\gamma \omega}
\end{array} \right.
\]

With the specification of the utility function in Eq. 20, we obtain directly the following result:

**Proposition 1 (Context-dependent substitutability)** With a CDS utility function,

- consumption and environmental quality are complements if and only if
\[
x^{\gamma + \alpha} y^{\omega + \beta} > \frac{\theta \alpha \beta}{\gamma \omega}.
\]

\(^{24}\) Two other possible functions are \( u(x, y) \equiv x^\gamma y^\omega + \theta \ln(x^\alpha + y^\beta) \) and \( u(x, y) \equiv xy - e^{\theta - x - y} \).

\(^{25}\) The preferences represented by \( u \) are not homothetic. The properties of function \( u \) are consistent with preferences satisfying completeness, continuity, transitivity, non-satiation.

\(^{26}\) See Appendix D for the first and second derivatives.
• consumption and environmental quality are substitutes if and only if
  \[ x^\gamma y^\omega + \beta < \frac{\alpha \beta}{\gamma \omega}. \]
• consumption and environmental quality are independent if and only if
  \[ x^\gamma y^\omega + \beta = \frac{\alpha \beta}{\gamma \omega}. \]

From these conditions, we can specify the function \( \Theta \) characterizing the contexts of substitutability in this example (Assumption 2):

\[
\Theta(x, y) \equiv x^\gamma y^\omega + \beta - \alpha \beta. 
\]

This \( \Theta \) function is an increasing function of \( x \) and \( y \). To better understand the meaning of the function \( \Theta \), we consider a definition of “wealth” which is more comprehensive than in the usual economic sense – referring economic-ecological wealth.

**Definition 4 (Economic-ecological wealth)** Economic-ecological wealth is measured by an increasing function of the economic and environmental endowments \( x, y \). In the CDS-utility case, we define economic-ecological wealth as the quantity \( x^\gamma y^\omega + \beta \).

The economic-ecological wealth considers a combination of consumption levels and environmental quality. The higher the value of this combination, the richer; the lower its value, the poorer.

Definition 4 allows us to distinguish two domains:

• *economic-ecological poverty* \( (x^\gamma y^\omega + \beta < \alpha \beta) \) is characterized by a negative cross derivative of the utility function, and corresponds to the domain where goods are considered as substitutes.
• *economic-ecological wealth* \( (x^\gamma y^\omega + \beta > \alpha \beta) \) is characterized by a positive cross derivative of the utility function, and corresponds to the domain where goods are considered as complements.

Figure 1 illustrates these two domains. The two domains \( (u_{xy} > 0 \text{ and } u_{xy} < 0) \) are separated by the independent context \( (u_{xy} = 0) \) represented by the red curve on Figure 1.

To study the impacts of this specification on environmental preferences and discounting, we derive the expressions of the elasticities and cross elasticities of marginal utility from our specific function:

\[
\eta_{x,x} = \frac{\gamma(1-\gamma)x^\gamma y^\omega + \beta}{x^\gamma y^\omega + \beta} > 0 \tag{22}
\]
\[
\eta_{y,y} = \frac{\omega(1-\omega)x^\gamma y^\omega + \beta}{x^\gamma y^\omega + \beta} > 0 \tag{23}
\]
\[
\eta_{x,y} = \frac{\alpha \beta - \gamma x^\gamma y^\omega}{x^\gamma y^\omega + \beta} \leq 0 \tag{24}
\]
\[
\eta_{y,x} = \frac{\alpha \beta - \gamma x^\gamma y^\omega}{x^\gamma y^\omega + \beta} \leq 0 \tag{25}
\]
As $u_x > 0$, $u_{xx} < 0$ and $u_y > 0$, $u_{yy} < 0$ $\forall x, y > 0$, $\eta_{x,x}$ and $\eta_{y,y}$ are positive everywhere and the higher $x$ and $y$ the lower they are. However, depending on the sign of $u_{xy}$ (i.e. depending on the context), $\eta_{x,y}$ and $\eta_{y,x}$ can be either positive or negative. But still, the bigger $x$ and $y$, the lower the two cross elasticities.\footnote{See Appendix E.}

4 Context-dependent environmental preferences

4.1 Environmental preferences and context-dependent substitutability

In subsection 2.2, we introduced the income and environmental elasticities of WTP and showed that they both depend on substitutability. The following Propositions 2 and 3 specify the way the elasticities of willingness-to-pay depend on the context in our theoretical framework.\footnote{Proofs are respectively in Appendices B and C.}

**Proposition 2 (context-dependent income elasticity of WTP)**  The income elasticity of willingness-to-pay is negative (resp. positive) if and only if $\eta_{y,x} > \eta_{x,x}$ (resp. $\eta_{y,x} < \eta_{x,x}$). Under Assumptions 1 and 2, a necessary condition for the former is that $\eta_{y,x} > 0$, i.e., the goods are substitutes.
The common case of positive income elasticity of WTP is not restrictive as long as $\eta_{y,x} < \eta_{x,x}$. This means that the WTP for environmental improvements of a decision-maker increases as her income grows whatever the context (substitutability or complementarity). However, it is possible that the decision-maker exhibits a negative income elasticity but only in contexts where goods are substitutes. This means that the decision-maker’s WTP decreases as her income grows. This case is underrepresented in empirical literature and often left out. May the context explain this "anomaly"? Such a decision-maker might already have a great environmental quality and not enough income to spend more on environmental issues. We discuss this situation in subsection 4.2 through the example of the CDS utility function.

**Proposition 3 (context-dependent environmental elasticity of WTP)** The environmental elasticity of willingness-to-pay is negative (resp. positive) if and only if $\eta_{x,y} < \eta_{y,y}$ (resp. $\eta_{x,y} > \eta_{y,y}$). Under Assumptions 1 and 2, a necessary condition for the latter is that $\eta_{x,y} > 0$, i.e., the goods are substitutes.

Intuitively, one might think that WTP should decrease when environmental quality improves (negative environmental elasticity of WTP). Proposition 3 confirms this intuition since no restrictive condition on the kind of relationship between goods is imposed for getting a negative environmental elasticity. But a positive environmental elasticity is a possible result when goods are considered as substitutes, which means that, as environmental quality grows, the decision-maker’s WTP increases too. One could imagine that this type of decision-maker have a high income but a rather poor environmental quality, and spend more money on environmental quality because she can afford it and the benefit in terms of well-being is positive.

Applying Propositions 2 and 3 to a classic (and widespread in the literature) isoelastic CES utility function as described in subsection 2.4 (Eq. 15) always results in a positive income elasticity and a negative environmental elasticity of WTP. This kind of utility function cannot explain the case of negative income effects on WTP found in the empirical literature. The next subsection analyzes the implications of the CDS function defined in subsection 3.2.

### 4.2 CDS utility function

The CDS function allows us to illustrate the results of Propositions 2 and 3. The sign of $\eta_{y,x} - \eta_{x,x}$ and $\eta_{x,y} - \eta_{y,y}$ can change according to the context for some parameters values, i.e. for some values of the weights attributed to the economy and the environment ($\gamma, \alpha, \omega, \beta$).

---

29 Since $\eta_{y,x} > \eta_{x,x} > 0$, which means that $u_{xy} < 0$ (domain of substitutability).
30 $\eta_{x,y} < \eta_{y,y}$ allows $\eta_{x,y}$, which is an indicator of substitutability between goods, to be either positive or negative.
31 Since $\eta_{x,y} > \eta_{y,y} > 0$, which means that $u_{xy} < 0$.
32 Since considering the same notations as in subsection 2.4, $\forall x, y > 0$, $\frac{\partial WTP}{\partial x} = \frac{1-b}{b} vy^{-\nu}x^{\nu-1} > 0$ and $\frac{\partial WTP}{\partial y} = -\frac{1-b}{b} vx^{-\nu}y^{\nu-1}x^{\nu} < 0$.

---

16
According to Proposition 2, if $\eta_{y,x} - \eta_{x,x} > 0$, the income elasticity of WTP is negative. Proposition 4 gives the conditions for which the income elasticity can be negative.\(^{33}\)

**Proposition 4** If \( \frac{\omega}{\beta\gamma} < \left(\frac{\gamma + \alpha - 1}{\gamma + \alpha + 1}\right)^2 \), there are contexts in which the income elasticity of WTP, $\varepsilon_{WTP}^x$, is negative.

In Figure 2, the dashed line represents the limit between the substitutability and the complementarity domains. The income elasticity is negative in contexts located between the red and the blue curves, and positive elsewhere. For some combinations of income and environmental quality, the decision-maker’s WTP for environmental quality decreases as income increases. In the substitutability domain, people with higher income give less value to the environment as income is a substitute for environmental quality. The restrictive condition \( \frac{\omega}{\beta\gamma} < \left(\frac{\gamma + \alpha - 1}{\gamma + \alpha + 1}\right)^2 \) is equivalent to \( \frac{\omega}{\gamma} < \frac{\beta}{\alpha} \) can be interpreted as follows: the relative weight for environmental quality is more important in the needs part of the CDS function than in the classic part.

**Proposition 5** If \( \frac{\beta\gamma}{\omega} < \left(\frac{\omega + \beta - 1}{\omega + \alpha + 1}\right)^2 \), then there are contexts in which the environmental elasticity of WTP, $\varepsilon_{WTP}^y$, is positive.

In Figure 3, the environmental elasticity of WTP is positive between the two curves. In this domain, the decision-maker is more willing to pay for environmental quality as this quality increases. Because such a decision-maker is in the substitutability domain, one can consider that since environmental quality grows and is a substitute for income,\(^{33}\) Proof in Appendix F
keeping income does not make more sense than assigning it to more environmental quality improvement. The restrictive condition \( \frac{\beta \gamma}{\alpha \omega} < \left( \omega + \beta - 1 \right)^2 \Rightarrow \frac{\beta \gamma}{\alpha \omega} < 1 \Leftrightarrow \frac{\beta}{\alpha} < \frac{\omega}{\gamma} \) implies that the relative weight of the environment is more important in the classic part of the CDS function than in its needs part.

From Propositions 4 and 5, we can prove the following proposition.

**Proposition 6 (Incompatibility of the two anomalies)** A decision-maker can never have both a negative income elasticity of WTP and a positive environmental elasticity of WTP.

Indeed, under Proposition 4, a negative income elasticity records \( \frac{\omega}{\gamma} < \frac{\beta}{\alpha} \) and under Proposition 5, a positive environmental elasticity records \( \frac{\beta}{\alpha} < \frac{\omega}{\gamma} \). These conditions are not compatible, hence Proposition 6. Both anomalies occur in the substitutability domain. Thus, under the utility maximization principle, the good whose quantity increases is the good which contributes more to well-being (either income or environmental quality). If it is environmental quality, then Proposition 5 applies.

## 5 Context-dependent discounting

In this section, we put forward the impact of the substitution effect on discounting.

### 5.1 Consequences of the substitution effect

In subsection 2.3, we showed that the economic discount rate \( r^x \) and the environmental discount rate \( r^y \) depend on substitutability between goods (via the so-called substitution
effect). Their expressions are reminded below:

\[
\begin{align*}
    r^x &= \delta + g_x \eta_{x,x} + g_y \eta_{x,y} \\
    r^y &= \delta + g_y \eta_{y,y} + g_x \eta_{y,x}
\end{align*}
\]

with \( g_x = \frac{\dot{x}}{x} \) and \( g_y = \frac{\dot{y}}{y} \).

The general theoretical framework proposed in Section 3 leads to distinct signs of the substitution effect whether the produced good and the environmental quality are substitutable or complementary,\(^{34}\) given growth prospects for consumption and environmental quality. Those are summarized in Table 1, which leads directly to Proposition 7.

### Table 1: Sign of the substitution effect

<table>
<thead>
<tr>
<th>Substitution Effect</th>
<th>Substitutability</th>
<th>Complementarity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r^x )</td>
<td>( g_y &gt; 0 )</td>
<td>-</td>
</tr>
<tr>
<td>( r^y )</td>
<td>( g_y &lt; 0 )</td>
<td>+</td>
</tr>
<tr>
<td>( g_x &gt; 0 )</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( g_x &lt; 0 )</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

**Proposition 7** The substitution effect has opposite effects on the two discount rates whether consumption and environmental quality are complements (\( \eta_{x,y} < 0 \) and \( \eta_{y,x} < 0 \)) or substitutes (\( \eta_{x,y} > 0 \) and \( \eta_{y,x} > 0 \)).

The substitution effect can either reduce or increase the economic (resp. ecological) discount rate due to the possibility of the two goods to be substitutes or complements. Our results are more general than the results obtained in the case of an isoelastic CES utility function. Indeed, Traeger (2011) showed (in his Proposition 1) that the substitution effect always increases the economic discount rate and always reduces the ecological discount rate. This is due to the assumption he makes on the kind of relationship between goods. Fixing the kind of relationship between the two components of the utility function determines exogenously the sign of the substitution effect,\(^{35}\) hence the discount rates, for a given growth prospect. As a consequence, only one line of Table 1 is obtained in the CES case. For example, in a context of uncertainty in his section 6.3, Gollier (2010) estimates the rate at which changes in biodiversity and consumption should be discounted. It is assumed that the intratemporal elasticity of substitution is 0.5, 1 or 1.5 (resp. resistance to intratemporal substitution is 2, 1 or \( \approx 0.7 \)) and the risk aversion is supposed to be 2.4. Thus for all the considered values of the elasticity of substitution, goods are substitutes in the Edgeworth-Pareto sense\(^{36}\) (see subsection 2.3). Same remark

\(^{34}\)When goods are independent, there is no substitution effect, only the growth effect influences the discount rates.

\(^{35}\)Kögel (2009) gives details on this specification.

\(^{36}\)Since \( 2 - 2.4 = -0.4 < 0 \), \( 1 - 2.4 = -1.4 < 0 \) and \( 0.7 - 2.4 = -1.7 < 0 \).
for Traeger (2011) in his example 2, for the two values of the intratemporal elasticity of substitution that he takes and the value he considers for the intertemporal elasticity of substitution, goods are substitutes in the E-P sense. Whatever the exogenous parameters (elasticities of substitution) supposed, only one kind of relationship between goods is considered in their model.

Within our framework, consider an economic growth ($g_x > 0$) and a depreciation of environmental quality ($g_y < 0$). In the Michel & Rotillon (1995) sense, complementarity between goods reflects a distaste effect: a decrease of environmental quality decreases the marginal utility for consumption. Substitutability reflects a compensation effect: a decrease in environmental quality increases the marginal utility for consumption. In the domain of complementarity, the economic substitution effect is positive (see Table 1) so it increases the economic discount rate $r^x$. Thus, the decision-maker is more concerned about the present than the future economic flows. This result comes from the distaste effect: since environmental quality will be lower in the future, and as environmental quality and consumption goods are complements, future utility will be reduced by environmental degradation. There is thus no reason to be very sensitive to future consumption changes and one is more prone to consume in the present. By contrast, the environmental substitution effect is negative and thus decreases the environmental discount rate $r^y$. This way, the decision-maker gives a high weight to future environmental quality because consumption is increasing and would result in a higher future utility if the environment quality is good, in complement to high income. Future environment is not discounted. In the domain of substitutability however, the substitution effect has opposite effects: $r^x$ decreases and $r^y$ increases. The decision-maker prefers to consume produced goods in the future than in the present because (i) environmental quality is diminishing and (ii) consumption will be the substitute for the loss of environmental quality. However, she discounts more environmental quality because consumption increases so there is no need in conserving environmental quality which can be substituted for by the increasing consumption.

5.2 CDS utility function: study of some interesting growth prospects

The benefits of a context-dependent substitutability are particularly well-illustrated when an ecological (or economic) shock occurs in that it is possible to move from one domain to another. Through the example of the CDS utility function, in Figure 4, an ecological (resp. economic) shock could change a decision-maker’s context from point $O_C$ (initial context in the domain of complementarity) to point $O_{S1}$ (resp. $O_{S2}$) located in the domain of substitutability. Such a shock changes the decision-maker’s behavior toward the environment and consumption. Thus, assessing a project before the shock or after it results in different conclusions, since the discount rates considered are different.

Next, we analyze some interesting growth schemes with the CDS utility function. We remind that not only the substitution effect (henceforth SE) depends on the good’s
quantities but also the growth effect\(^\text{37}\) (henceforth GE), since \(\frac{\partial \eta_{x,x}}{\partial x} < 0\), \(\frac{\partial \eta_{x,x}}{\partial y} < 0\), \(\frac{\partial \eta_{y,x}}{\partial x} < 0\) and \(\frac{\partial \eta_{y,y}}{\partial y} < 0\).\(^\text{38}\) This means that the elasticities of marginal utility for consumption and environmental quality increase as consumption and environmental quality levels decrease. More accurately, the growth effect is proportional to \(\eta_{x,x}\) for \(r^x\) and to \(\eta_{y,y}\) for \(r^y\) which have different magnitudes depending on the \(x\) and \(y\) levels. In our example, an economic-ecologically rich decision-maker has lower elasticities of marginal utility than a poor one.\(^\text{39}\) Thus, the magnitude of the growth effect is greater for economic-ecologically poor decision-makers.

Following Gollier (2010), we assume that \(\delta = 0\), both for intergenerational equity and to ease the discussion on the sign of the discount rates.

Discounting in a growing economy with environmental degradation Observing Table 2, a rich decision-maker (distaste effect) discounts a lot consumption and discounts negatively environmental quality. We can give the same explanation as in subsection 5.1: a rich decision-maker prefers present consumption over future consumption because future environmental quality will be lower and will undermine utility from consumption. For the

\(^{37}\)For both discount rates, the sign of the growth effect is the same as the sign of the growth rate.

\(^{38}\)Proof in Appendix E

\(^{39}\)When we talk about poor or rich decision-makers in this subsection, we refer to the economic-ecological wealth definition.
same reason, future environmental quality matters a lot (low ecological discount rate) because the economy will be better in the future ($g_x > 0$). For poor decision-makers, the sign of the discount rates are ambiguous as the GE and SE have of opposite effects.

**Discounting in a growing economy with environmental improvements** According to Table 3, a poor decision-maker has very high discount rates. It can be attributed to a compensation effect, even a double compensation effect: since consumption grows, there is no need in conserving environmental quality, hence a high environmental discount rate. And since environmental quality also rises, then it is not important to save money for future consumption, hence a high consumption discount rate.

**Discounting in a recessive economy with environmental degradation** From Table 4, we can say that in a recessive situation, a rich decision-maker discounts consumption and environmental quality more than a poor decision-maker. A poor decision-maker has very low discount rates. As both the sources of utility are declining, this is not possible for a poor to substitute one good for the decline of the other. Future looks terrible and the decision-maker is ready to “save” at a low discount rate even if she is not endowed a lot. The growth and substitution effects mitigate each other for the rich: as the two goods are complements, having one of them declining or both does not change things a lot.
Table 5: $g_x < 0 \& g_y > 0$

<table>
<thead>
<tr>
<th></th>
<th>$r_x$</th>
<th>$r_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Richs</td>
<td>GE SE</td>
<td>GE SE</td>
</tr>
<tr>
<td>Poors</td>
<td>– + ++</td>
<td>– + ++</td>
</tr>
</tbody>
</table>

Discounting in a recessive economy with environmental improvements. Observing Table 5, we can say that riches care a lot for future economic outcomes (low economic discount rate) and less for future environmental outcomes (higher ecological discount rate) since consumption decreases and environmental quality increases. An economic project with future economic benefits and environmental costs is likely to be accepted, at the margin of the given growth prospect. This case is ambiguous for a poor decision-maker, as the two effects have opposite directions.

6 Some suggestions for empirical testing of our theory

We suggest some empirical testing that could help to better understand the way people behave in different contexts.

6.1 Willingness-to-pay and the context

Negative income effects have been found in empirical literature (McFadden & Leonard, 1993, Horowitz, 2002, Huhtala, 2010). Now, the question is really why this occurs and especially if the decision-maker actually does consider the two goods as substitutes, as Proposition 2 puts forward.

Our conclusions about the impact of substitutability between goods on the variations of WTP are partly in line with Shogren et al. (1994). They test Hanemann (1991)'s proposition that the divergence between WTP and WTA (willingness-to-accept) is driven by the degree of substitution between goods. They conclude that the more substitutable the goods, the smaller the divergence between WTA and WTP ($\frac{WTA}{WTP} \to 1$). Our conclusions differ in that they suppose a positive income elasticity of WTP whereas we showed that a negative one is possible in particular contexts. Therefore, an interesting empirical research challenge would be to find out in which contexts the income and environmental elasticities of WTP are positive or negative in order to test Proposition 2 and Proposition 3.

Consider for example different countries (or regions within a country) which could be sorted according to population’s income level (via GDP or household income) and environmental quality (environmental indicators such as air or water quality), and then be classified into two groups, i.e., economic-ecological wealth vs. poverty. Then we could

---

As a reminder, a ratio $\frac{WTA}{WTP} < 1$ indicates a negative income effect on WTP (Horowitz & McConnell, 2003) and a ratio $\frac{WTA}{WTP} > 1$ is implied by a positive income effect on WTP.

Which is actually an assumption of Hanemann’s proposition.
analyze the variations of WTP with income and environmental quality and determine in which contexts they are positive and negative. Such an empirical analysis or experiment would be lead in a framework where wealth is a wider concept than usually since it considers both economic wealth and ecological wealth. This would allow a comparison with empirical literature at least on income effects on WTP; we would see if and how our framework fits empirical data.

6.2 Discounting and the context

Similarly, some empirical testing might disentangle the effects of each relationship between goods on the discount rates’ values: in which contexts are the economic and the environmental discount rates rather high or low? By considering the same kind of sorting as in subsection 6.1, we could analyze the different values of the discount rates in each country (or region of a country) and see if our specification fits the results better than usual specifications such as isoelastic CES utility functions.

7 Conclusions

The answer to the question “are consumption of man-made goods and environmental quality complements or substitutes?” is “it should depend on the context.” Choices and stated preference are contingent on consumption level and environmental quality, and on the substitutability between these sources of utility.

In this paper, we showed that the kind of relationship between goods (substitutability, complementarity or independence) is important in determining (i) WTP changes with goods’ quantities (income, environmental quality) and (ii) the discount rates’ values. In classical models, this kind of relationship is fixed through the choice of exogenous parameters such as the intertemporal and intratemporal elasticities of substitution, and not linked to levels of consumption and environmental quality at all. This leads to the implicit determination of discount rates especially and is not without consequences for the interpretation of the decision-maker’s behavior. Our framework in contrast, allows the relationship between goods to depend on the quantities of the goods. This affects the changes of WTP with the good’s quantities. In particular, we put forward two “anomalies”: WTP for environmental improvements can decrease with income and increase with environmental quality in the domain of substitutability between goods. It could partly explain some empirical results which are underrepresented (negative income elasticities of WTP) and calls for more research. Furthermore, the kind of relationship between goods determines the way the economic and environmental growth rates impact the discount rates. Thus in our framework, the discount rates vary with the kind of relationship between goods implied by the goods’ quantities, through the substitution effect. The interest of the framework we develop is particularly put forward when studying an exogenous (ecological or economic) shock since it can change the way the decision-maker behaves toward the environment and consumption. A shock is reflected in different discount rates’ values because the sign of the substitution effect can change, depending
on the magnitude of the shock. Therefore, an avenue for research would be to analyze how the preferences we defined in Section 3 change the growth path in classical natural resources models considering environmental quality as a contributor to well-being. This way, we could determine optimal growth rates and study discount rates’ evolution over time and particularly exogenous shocks on goods’ quantities. It could also be interesting to study an environmental Kuznets curve trajectory and derive the evolution of the discount rates along this path.

Another interesting further research would be to run an experimental project in order to test (i) the link between substitutability and WTP changes with income and environmental quality and (ii) the link between substitutability and discount rates’ values.

Appendices

A Discount rates’ expressions

Let the present value of utility at time \( t \) be denoted by \( U(x(t), y(t), t) = u(x(t), y(t))e^{-\delta t} \). The marginal utility derived from an increment of good \( x \) at time \( t \) is

\[
U_x(x(t), y(t), t) = u_x(x(t), y(t))e^{-\delta t},
\]

(26)

By definition, the consumption discount rate, \( r^x \) is the rate of change of marginal utility for consumption as time changes:

\[
r^x \equiv -\frac{\dot{U}_x(x(t), y(t), t)}{U_x(x(t), y(t), t)} = \frac{1}{u_x(x(t), y(t))e^{-\delta t}} \frac{du_x(x(t), y(t))e^{-\delta t}}{dt}
\]

(27)

As

\[
\dot{U}_x(x(t), y(t), t) = \frac{du_x(x(t), y(t))e^{-\delta t}}{dt} = (\dot{x} u_{xx} + \dot{y} u_{xy}) e^{-\delta t} - u_x \delta e^{-\delta t},
\]

(28)

we get

\[
r^x = -\frac{\dot{x} u_{xx} + \dot{y} u_{xy}}{u_x} - \frac{u_x \delta}{u_x} = \delta - \frac{\dot{x}}{x} \left( \frac{u_{xx}}{u_x} \right) - \frac{\dot{y}}{y} \left( \frac{u_{xy}}{u_x} \right)
\]

(29)

Let \( \eta_{x,x} = -\frac{u_{xx}}{u_x} \) be the elasticity of marginal utility of consumption with respect to consumption and \( \eta_{x,y} = -\frac{u_{xy}}{u_x} \) be the elasticity of marginal utility of consumption with respect to environmental quality, then:

\[
r^x = \delta + \frac{\dot{x}}{x} \eta_{x,x} + \frac{\dot{y}}{y} \eta_{x,y}.
\]

(30)

Now the same can be done for an increment of environmental good \( y \). The environmental or ecological discount rate, \( r^y \) is the rate of change of marginal utility for environmental quality as time changes

\[
r^y \equiv -\frac{\dot{U}_y(x(t), y(t), t)}{U_y(x(t), y(t), t)} = \frac{1}{u_y(x(t), y(t))e^{-\delta t}} \frac{du_y(x(t), y(t))e^{-\delta t}}{dt}
\]

(31)

The negative sign is attributed to the fact that the discount rate is conventionally positive whereas the discount factor growth rate is negative (negative slope).

42The negative sign is attributed to the fact that the discount rate is conventionally positive whereas the discount factor growth rate is negative (negative slope).
As
\[
\dot{U}_y(x(t), y(t), t) = \frac{du_y(x(t), y(t))e^{-\delta t}}{dt} = (\dot{y}u_{yy} + \dot{x}u_{yx})e^{-\delta t} - u_y\delta e^{-\delta t},
\]
we get
\[
\dot{r}_y = -\left[\frac{\dot{y}u_{yy} + \dot{x}u_{yx}}{u_y}\right] - u_y\delta = \delta - \frac{\dot{y}}{y} \left(\frac{u_{yy}}{u_y}\right) - \frac{\dot{x}}{x} \left(\frac{u_{yx}}{u_y}\right).
\]
Let \(\eta_{y,y} = -\frac{u_{yy}}{u_y}\) be the elasticity of marginal utility of environmental quality with respect to environmental quality and \(\eta_{y,x} = -\frac{xu_{yx}}{u_y}\) the elasticity of marginal utility of environmental quality with respect to consumption, then:
\[
\dot{r}_y = \delta + \frac{\dot{y}}{y} \eta_{y,y} + \frac{\dot{x}}{x} \eta_{y,x}
\]

\section*{B Proof of Proposition 2}

We remind that \(WTP = \frac{w}{w_x}\) and that \(\varepsilon_{WTP}^x = \frac{\partial WTP}{\partial x}\). Then the sign of \(\varepsilon_{WTP}^x\) is the same as the sign of \(\frac{\partial WTP}{\partial x}\).
\[
\varepsilon_{WTP}^x > 0 \iff \frac{\partial WTP}{\partial x} > 0 \tag{35}
\\iff u_{xy}u_x - u_{xx}u_y > 0 \tag{36}
\\iff u_{xy} > \frac{u_{xx}}{u_x} \tag{37}
\\iff -\frac{xu_{xy}}{u_y} < \frac{-xu_{xx}}{u_x} \tag{38}
\\iff \eta_{y,x} < \eta_{x,x} \tag{39}
\varepsilon_{WTP}^x < 0 \iff \eta_{y,x} > \eta_{x,x} \tag{40}
\]

\section*{C Proof of Proposition 3}

We remind that \(WTP = \frac{w}{w_y}\) and that \(\varepsilon_{WTP}^y = \frac{\partial WTP}{\partial y}\). Then the sign of \(\varepsilon_{WTP}^y\) is the same as the sign of \(\frac{\partial WTP}{\partial y}\).
\[
\varepsilon_{WTP}^y > 0 \iff \frac{\partial WTP}{\partial y} > 0 \tag{41}
\\iff u_{yy}u_x - u_{xy}u_y > 0 \tag{42}
\\iff u_{yy} > \frac{u_{xy}}{u_y} \tag{43}
\\iff -\frac{y}{u_y} \frac{u_{yy}}{u_y} < \frac{-y}{u_x} \frac{u_{xy}}{u_x} \tag{44}
\\iff \eta_{y,y} < \eta_{x,y} \tag{45}
\varepsilon_{WTP}^y < 0 \iff \eta_{y,y} > \eta_{x,y} \tag{46}
\]
D  First and second partial derivatives of the CDS utility function

First derivatives

\[ u_x = \gamma x^{\gamma-1}y^{-\beta} + \theta \alpha x^{-\alpha -1}y^{-\beta} > 0 \quad \forall \ x, y > 0 \]  \hspace{1cm} (47)

\[ u_y = \omega x^{\gamma}y^{\omega-1} + \theta \beta x^{-\alpha}y^{-\beta -1} > 0 \quad \forall \ x, y > 0 \]  \hspace{1cm} (48)

Second derivatives

\[ u_{xx} = \gamma(\gamma - 1)x^{\gamma-2}y^{2} - \theta \alpha (\alpha + 1)x^{-\alpha -2}y^{-\beta} < 0 \quad \forall \ x, y > 0 \]  \hspace{1cm} (49)

\[ u_{yy} = \omega(\omega - 1)x^{\gamma}y^{\omega-2} - \theta \beta (\beta + 1)x^{-\alpha}y^{-\beta -2} < 0 \quad \forall \ x, y > 0 \]  \hspace{1cm} (50)

E  Variations of the elasticities and cross elasticities of marginal utility with income and environmental quality

\[ \frac{\partial \eta_{x,x}}{\partial x} < 0 \iff 1 - \gamma < \alpha + 1 \]  \hspace{1cm} & \hspace{1cm} \frac{\partial \eta_{x,x}}{\partial y} < 0 \iff 1 - \gamma < \alpha + 1 \]

\[ \frac{\partial \eta_{x,y}}{\partial x} < 0 \iff -\omega < \beta \]  \hspace{1cm} & \hspace{1cm} \frac{\partial \eta_{x,y}}{\partial y} < 0 \iff -\omega < \beta \]

\[ \frac{\partial \eta_{y,y}}{\partial x} < 0 \iff -\gamma < \alpha \]  \hspace{1cm} & \hspace{1cm} \frac{\partial \eta_{y,y}}{\partial y} < 0 \iff -\gamma < \alpha \]

All these conditions are always true because \( 0 < \gamma, \alpha, \omega, \beta < 1 \).

F  Proof of Proposition 4

The sign of the income elasticity of WTP is opposite to the sign of the difference between \( \eta_{y,x} \) and \( \eta_{x,x} \).

\[ \eta_{y,x} - \eta_{x,x} > 0 \iff \frac{\theta \alpha \beta - \gamma \omega x^{\gamma + \alpha} y^{\omega + \beta}}{\omega x^{\gamma + \alpha} y^{\omega + \beta} + \theta \beta} - \frac{\gamma(1 - \gamma)x^{\gamma + \alpha} y^{\omega + \beta} + \theta \alpha (\alpha + 1)}{\gamma x^{\gamma + \alpha} y^{\omega + \beta} + \theta \alpha} > 0 \]  \hspace{1cm} (51)

\[ \iff Z^2(\gamma \omega) + Z(\theta \alpha \beta \gamma - \gamma \omega \alpha - \gamma(1 - \gamma)\beta - \theta \alpha \omega (\alpha + 1)) - \theta^2 \alpha \beta > 0 \]  \hspace{1cm} (52)

\[ \iff AZ^2 + BZ + C > 0 \]  \hspace{1cm} (53)

with

\[
\begin{align*}
Z &= x^{\gamma + \alpha} y^{\omega + \beta} \\
A &= -\gamma \omega < 0 \\
B &= \theta(\beta \gamma (\alpha + \gamma - 1) - \alpha \omega (\alpha + \gamma + 1)) \leq 0? \\
C &= -\theta^2 \alpha \beta < 0 
\end{align*}
\]

The discriminant of the polynomial \( P(Z) = AZ^2 + BZ + C \) is:

\[ \Delta = \theta^2 (\beta \gamma - \alpha \omega) [\beta \gamma (\gamma + \alpha - 1)^2 - \alpha \omega (\gamma + \alpha + 1)^2] \]  \hspace{1cm} (54)

\( \Delta \) is positive if and only if

\[
\begin{align*}
(1) \left\{ \begin{array}{l}
\beta \gamma - \alpha \omega > 0 \\
\beta \gamma (\gamma + \alpha - 1)^2 - \alpha \omega (\gamma + \alpha + 1)^2 > 0
\end{array} \right.
\end{align*}
\]
or

\[
\begin{align*}
\text{(2)} \quad & \begin{cases}
\beta \gamma - \alpha \omega < 0 \\
\beta \gamma (\gamma + \alpha - 1)^2 - \alpha \omega (\gamma + \alpha + 1)^2 < 0
\end{cases}
\end{align*}
\]

When \( \Delta > 0 \), the two resulting real roots \( Z_1 = \frac{-B + \sqrt{\Delta}}{2A} \) and \( Z_2 = \frac{-B - \sqrt{\Delta}}{2A} \) are:

\[
Z_1 = \frac{4 \theta \beta \gamma \alpha \omega}{\gamma \omega [\beta \gamma (\gamma + \alpha - 1) - \alpha \omega (\gamma + \alpha + 1) + \sqrt{(\beta \gamma - \alpha \omega)(\beta \gamma (\gamma + \alpha - 1)^2 - \alpha \omega (\gamma + \alpha + 1)^2)}]}
\]

\[
Z_2 = \frac{-4 \theta \beta \gamma \alpha \omega}{\gamma \omega [\alpha \omega (\gamma + \alpha + 1) - \beta \gamma (\gamma + \alpha - 1) + \sqrt{(\beta \gamma - \alpha \omega)(\beta \gamma (\gamma + \alpha - 1)^2 - \alpha \omega (\gamma + \alpha + 1)^2)}]}
\]

The sign of the two roots depends on whether condition (1) or condition (2) hold:

- if condition (1) holds, as \( \frac{\alpha \omega}{\beta \gamma} < \frac{(\gamma + \alpha - 1)^2}{(\gamma + \alpha + 1)^2} \) \( \Leftrightarrow \frac{\alpha \omega}{\beta \gamma} < \frac{\gamma + \alpha - 1}{\gamma + \alpha + 1} \) because \( \frac{\gamma + \alpha - 1}{\gamma + \alpha + 1} < 1 \), then \( B > 0 \) and:

\[
\begin{cases}
Z_1 > 0 \text{ since } \frac{\alpha \omega}{\beta \gamma} < \frac{\gamma + \alpha - 1}{\gamma + \alpha + 1} \\
Z_2 > 0
\end{cases}
\]

- if condition (2) holds then as \( B < 0 \):

\[
\begin{cases}
Z_1 < 0 \text{ since } \frac{\alpha \omega}{\beta \gamma} > \frac{\gamma + \alpha - 1}{\gamma + \alpha + 1} \\
Z_2 < 0
\end{cases}
\]

Only condition (1) is interesting for us to get a negative income elasticity. Then this income elasticity is negative between the curves of equation \( Z = Z_1 \) and \( Z = Z_2 \). Everywhere else, the income elasticity of WTP is positive.

**G Proof of Proposition 5**

The sign of the environmental elasticity of WTP is the same as the sign of the difference between \( \eta_{x,y} \) and \( \eta_{y,y} \).

\[
\eta_{x,y} - \eta_{y,y} > 0 \Leftrightarrow \frac{\theta \alpha \beta - \gamma \omega x^{\gamma + \alpha} y^w + \beta}{\gamma x^{\gamma + \alpha} y^w + \beta} - \frac{\omega (1 - \omega) x^{\gamma + \alpha} y^w + \beta}{\omega x^{\gamma + \alpha} y^w + \beta} > 0
\]

\[
\Leftrightarrow Z^2 (-\gamma \omega) + Z(\theta \alpha \beta - \theta \gamma \omega \beta - \theta \alpha \omega (1 - \omega) - \theta \gamma (\beta + 1)) - \theta^2 \alpha \beta > 0
\]

\[
\Leftrightarrow AZ^2 + BZ + C > 0
\]

with

\[
\begin{align*}
Z &= x^{\gamma + \alpha} y^w + \beta \\
A &= -\gamma \omega < 0 \\
B &= \theta (\alpha \omega (\beta + \omega - 1) - \beta \gamma (\beta + \omega + 1)) \Leftrightarrow 0? \\
C &= -\theta^2 \alpha \beta < 0
\end{align*}
\]

The discriminant of the polynomial \( P(Z) = AZ^2 + BZ + C \) is:

\[
\Delta = \theta^2 [(\alpha \omega - \beta \gamma) [\alpha \omega (\omega + \beta - 1)^2 - \beta \gamma (\omega + \beta + 1)^2]]
\]

\( \Delta \) is positive if and only if

\[
\begin{align*}
\text{(3)} \quad & \begin{cases}
\alpha \omega - \beta \gamma > 0 \\
\alpha \omega (\beta + \omega - 1)^2 - \beta \gamma (\beta + \omega + 1)^2 > 0
\end{cases}
\end{align*}
\]
or
\[
(4) \begin{cases} \alpha \omega - \beta \gamma < 0 \\
\alpha \omega (\beta + \omega - 1)^2 - \beta \gamma (\beta + \omega + 1)^2 < 0 \end{cases}
\]

For $\Delta > 0$, the two resulting real roots $Z_1$ and $Z_2$ are:

\[
Z_1 = \frac{4\beta \gamma \alpha \omega}{\gamma \omega (\alpha \omega (\beta + 1) - \beta \gamma (\alpha \omega (\omega + 1) + \sqrt{(\alpha \omega - \beta \gamma)(\alpha \omega (\omega + 1) - \beta \gamma (\omega + 1)^2})}}
\]

\[
Z_2 = \frac{-4\beta \gamma \alpha \omega}{\gamma \omega (\beta \gamma (\omega + 1) - \alpha \omega (\omega + 1) - \sqrt{(\alpha \omega - \beta \gamma)(\alpha \omega (\omega + 1) - \beta \gamma (\omega + 1)^2})}}
\]

The sign of the two roots depends on whether condition (3) or condition (4) hold:

- if condition (3) holds:
  \[
  \begin{cases} Z_1 > 0 \text{ since } \frac{\beta \gamma}{\alpha \omega} < \frac{\omega + \beta - 1}{\omega + \beta + 1} \\
  Z_2 > 0
  \end{cases}
  \]

- if condition (4) holds:
  \[
  \begin{cases} Z_1 < 0 \text{ since } \frac{\beta \gamma}{\alpha \omega} > \frac{\omega + \beta - 1}{\omega + \beta + 1} \\
  Z_2 < 0
  \end{cases}
  \]

Only condition (3) is interesting for us to get a positive environmental elasticity. Then this elasticity is positive between the curves of equation $Z = Z_1$ and $Z = Z_2$. Everywhere else, the environmental elasticity of WTP is negative.

**References**


Ready, R, & Navrud, S. 2006. International benefit transfer: Methods and validity tests. *Eco-

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