Energy transition under the possibility of a stochastic catastrophe: a two-sector approach

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Abstract

In the present paper, we investigate the optimal energy transition of a two-sector economy with exhaustible oil reserves, a renewable source of energy and a stochastic critical pollution threshold above which a catastrophic event (following flooding for instance) may occur. In addition to the available Non Renewable Energy (NRE), part of capital is devoted to producing renewable energy (RE). Part of the total energy is used as energy services by a representative consumer through a quadratic utility function and the other part is used as input in a Leontief production function to produce final goods. The optimal energy transition path corresponds to three main regimes that may occur in the economy. In the first one, energy is produced by both oil and the renewable resource that are complementary and the pollution does not exceed its threshold level. At that time, the economy faces a risk of catastrophic events which occurrence is uncertain but it is more likely to happen in the case the economy faces an increasing pollution. After the catastrophic event has occurred, the economy switches to the second regime in which both energy sources are still used but it has already crossed the critical pollution threshold. During this second regime, the economy starts facing quadratic damages through a loss in utility. Only renewable resources are still used in the third regime but the economy is still facing the negative consequences of the catastrophic event.

We backward solve the model by using the optimality conditions and numerical simulations. Numerical results show that there exist reasonable parameter values that confirm the existence of the three phases of energy transition described above. It also leads to a conservative behaviour of the resource extraction so that the pollution level is lower than that of the deterministic threshold. We find two main effects of uncertainty: (i) the deterministic case does not yield an optimal transition path and (ii) for low uncertainty, the optimal solution may result in high use of the polluting resources and then more pollution is accumulated in the environment. One can conclude that uncertainty may favour the energy transition in the sense that people may fear about the future and make precautionary decisions.

Keywords: *energy, pollution, irreversibility, catastrophe, switch.*

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1 Introduction

Despite the growing investments in Renewable Energy (RE) production over year (63 to 244 billion USD from 2006 to 2012 (GEA,2013)), the use of fossil fuels as dirty energy is still the main energy source (78.2%) in the world. Energy transition involves two kinds of environmental concerns. First, the Non Renewable Energy (NRE) (for instance oil) is exhaustible and its use has negative externalities through environmental damages (for instance the occurrence of catastrophic event). One also need to adopt saving energy technologies in order to reduce energy use.

Uncertain catastrophe can be related to climate change issues as it is the most important uncertainty aspects of climate change from an economist's point of view (Gjerde et al., 1998). Under uncertainty on the occurrence of the catastrophic event in a two-sector economy, (i) how should the economy pollutes and invest in the Renewable Energy (RE)? and (ii) what is the effect of the uncertainty on the pollution threshold? In the present paper, we analyze the optimal energy transition of a two-sector economy with exhaustible oil reserves, a renewable source of energy and a pollution threat. The latter corresponds to a stochastic critical pollution threshold above which a catastrophic event (following flooding for instance) may occur. The economy produces energy (Oil and RE) and final goods. The oil extraction is costless while the renewable energy is produced by using a part of capital. Part of the energy is used as energy services by a representative consumer through a quadratic utility function and the other part is used as input in a Leontief production function to produce final goods. The use of the NRE and that of the RE are complementary. Numerical results show that the optimal energy transition path may correspond to three phases starting with both use of the energy resources that is followed by the catastrophe and finally by the only use of the RE. Uncertainty on the occurrence of the catastrophe induces a conservative behaviour and negatively affects the rate of the polluting resource extraction.

There exists a huge literature on the long run depletion of oil reserves (Dasgupta and Heal, 1974, 1979; Dasgupta and Stiglitz, 1981 and Krautkraemer, 1986), the polluting feature of the non-renewable resource (Nordhaus, 1994 and Tahvonen, 1996, 1997) and the irreversibility of the environmental damage. The latter can be (i) an exhaustion/decrease of the natural capacity of regeneration (Forster, 1975; Tavhonen and Withagen, 1996; Prieur, 2008; Pindyck, 2002; Fisher and Narain, 2003 and Ayong Le Kama et al., 2011) and (ii) an irreversibility in the decision process (Kolstad, 1996; Ulph and Ulph, 1997; Pommeret and Prieur, 2009). It can also be (iii) a ceiling on the pollution stock (Chakravorty et al., 2006, 2008, 2012 and Lafforgue et al., 2008), or (iv) a combination (Prieur et al., 2013). Moreover, Pommeret and Prieur (2009) consider a double irreversibility as a sunk cost of environmental degradation and a sunk cost of environmental policy (for instance, irreversible investments made to reduce GHG emissions) and show that the absence of irreversibilities may lead to too much pollution. On contrary, Kollenbach (2013) considers a reversible ceiling on the stock of pollution and shows that the ceiling increases the scarcity of the exhaustible resource in short run while there is no change in the long run.

In the literature, the damage function itself can be modelled as : Income loss (Tsur and Withagen, 2013), capital loss (Horii and Ikefuji, 2010), damages in production

(Golosov et al., 2011) or damages on the social welfare (Van der ploeg and Withagen, 2012). For instance, Van der ploeg and Withagen (2012) investigate the optimal climate policy in a green Ramsey growth model with exhaustible oil reserves, an infinitely elastic supply of renewable, stock-dependent oil extraction costs and convex climate damages on social welfare to find four regimes that depend on the initial social cost of oil and that of renewable.

Catastrophe is related to climate change issues as it is the most important uncertainty aspects of climate change from an economist's point of view (Gjerde et al., 1999). The issue of the catastrophic environmental event goes back to Cropper (1976) and Heal (1984). Cropper (1976) distinguishes two kinds of consequences of the catastrophe: a temporary reduction in utility and an irreversible damage. Both happen at the time the economy crosses an uncertain critical threshold (Pollution or extraction of resource) and it positively (resp. negatively) depends on the stock of pollution (resp. resource). Heal (1984) uses the same model and focuses on the catastrophe that is related to global warming where the atmosphere is favourable or unfavourable to the production. Since then, Various works have been done in economics that deal with the theory of catastrophe either in the same vein with Cropper (1976) (Clarke and Reed, 1994; Aronsson et al., 1998; Tsur and Zemel, 1998; Horii and Ikefuji, 2010.) or similarly to Heal (1984) (Tsur and Zemel, 1996, 2009; Gjerde et al., 1999; Mastrandrea and Scheinder, 2001; Naevdal, 2006; Haurie and Moreisino, 2006; Stern, 2007; Bahn et al., 2008; Weitzman, 2009; Tsur and Withagen, 2013).

For instance, Tsur and Zemel (1996) in the same vein with Heal (1984) studies an optimal management of atmospheric pollution under uncertainty on the occurrence of undesirable events related to greenhouse effect. He focuses mainly on the ignorance of the exact critical pollution threshold above which the catastrophe may occur. While, Gjerde et al., (1999) analyzes the optimal emission of greenhouse gases under a risk of irreversible global catastrophe by using simulations on an integrated assessment model to establish numerical model. Leizarowitz and Tsur (2012) study the optimal management of the renewable resource under the uncertainty of the event occurrence that is the result of two uncertain components: uncertain extreme environmental conditions and stochastic state of the resource. Under a risk of potential regime shifts, Polasky et al. (2011) find a precautionary optimal management of resource with higher resource stock levels. Under the risk of an abrupt and random jump in the damage function, Zeeuw and Zemel (2012) find the same precautionary behaviour. On contrary, Brozović and Schlenker (2011) find that a reduction in uncertainty can first increase and then decrease optimal precautionary activity. Ren and Polasky (2014) also find the same controversial result and show that there exists reasonable parameter values that lead to aggressive management.

Energy transition involves at least three regime switching that correspond to the thresholds on pollution and energy sources. In their recent paper, Boucekkine et al. (2013) provide in a deterministic framework, first order optimality conditions in an optimal regime switching problem with threshold effects, as an alternative to the widely used Bellman method. These optimality conditions are the continuity of appropriate co-states and states variables and that of the Hamiltonian. They apply this theory to the optimal management of exhaustible resources under ecological irreversibility and backstop adoption. We use the same optimality conditions to determine the optimal switching time to the full adoption of the RE and the level of the pollution. Several studies on the adoption of renewable energy have been done with the assumptions of imperfect or perfect substitution between inputs, but very few works focus on the complementarity between capital and energy (Pindyck and Rotemberg, 1983; Boucekkine and Pommeret, 2004; Diaz and Puch, 2013) and that of the NRE and RE (Pelli, 2012). The present paper fills this gap in the literature. Another novelty in our paper is the use of a two-sector approach to model the energy transition issue. To our knowledge, only Bahini and Le Van (2013) analyzes energy transition with a two-sector approach (energy sector and final good sector).

The remainder of this paper is structured as follows. The model is presented in section 2. We analyze the optimal energy transition in section 3. Section 4 is devoted to numerical results and sensitivity analysis. Section 5 discusses the difference between certainty and uncertainty pollution threshold. Finally, we conclude in section 6.

2 Model

We consider a closed economy with two productive sectors which we extend to Environment in a general equilibrium setting. In this two-sector economy, we consider an exhaustible oil reserves, a renewable source of energy and a pollution threat. The economy produces energy (oil extraction and renewable energy) and final goods. Part of the energy is used as energy services by a representative consumer through a quadratic utility function and the other part is used as input in a Leontief production function to produce final goods. The use of dirty energy by both final goods sector and households has negative impact on environment through a stochastic critical pollution threshold above which a catastrophic event (following flooding for instance) may occur. This uncertain catastrophic event has a damage on households that we assume quadratic and induces a loss in utility. In the following sections, we describe the energy sector, the final good sector, households' utility and the pollution threat respectively.

2.1 Energy sector

We consider energy as intermediate goods that has two main sources: oil as a non renewable energy (NRE) or "dirty energy" E_s and a renewable energy source (RE) as "clean energy" E_x. Both final goods sector and households use the energy. The former uses a part E_1 as input to produce final goods, while the latter uses a part of the energy E_2 as energy services. E_{2s}, E_{2x}, E_{1s} and E_{1x} are the part of the NRE and that of the RE uses of households and final goods sector respectively.

We suppose a costless NRE that corresponds to an extraction rate *s(t)* of the oil stock S_t generated at each time t by the following dynamics:

$$dS_t = -s(t)dt \tag{1}$$

where $E_{st} = s(t)$.

The renewable energy is produced by a "one-to-one" transformation of a part of capital (K_1) as follows:

$$E_x = K_1 \tag{2}$$

We neglect the environmental impact of the energy production and suppose that pollution is only due to the use of the dirty energy from households and final goods sector. We have the following energy market clearing conditions:

$$E_{st} = E_{1st} + E_{2st} \tag{3}$$

$$E_{xt} = E_{1xt} + E_{2xt} \tag{4}$$

2.2 Final good sector

Part E_2 of energy and that of capital (K_2) serve as inputs in a Leontief production function[1] to produce final goods Y. We assume that both oil and the renewable resource use are complementary[2].

$$Y = min\{K_2, E_{1t}\} \tag{5}$$

With $E_{1t} = min\{E_{1st}, E_{1xt}\}$ and $K_1 + K_2 = K$.

The final good Y can be used as capital good (for investment) or consumption goods:

$$Y = C + I$$

Where I in the total investment in energy sector and final goods.

2.3 Households

We consider a representative household who consumes the energy good E_2 as energy services and the non-energy good C. The consumption of the final good and the energy use are complementary[3]. We assume a quadratic utility function due to quadratic damage function and the Consumer's preferences are represented by the utility function U as an expected discounted sum of instantaneous utility flow u:

[1]Pindyck and Rotemberg (1983), Boucekkine and Pommeret (2004) and Diaz and Puch (2013) argue that there exists a complementarity between capital and energy services in final goods production. For instance, industrial production necessary requires at least capital and energy.

[2]Pelli (2012) proves with econometrics results the existence of complementarity between the NRE and RE. We mainly have in mind that the production of RE (for instance Solar panels) requires the NRE. The presence of rigidity in macroeconomic view can also justify this complementary between the NRE and RE.

[3]Mostly, households need energy to cook foods, to use electronic appliances or to use a car so that energy is complementary to final goods consumption in order to get utility or satisfaction.

$$U = \int_{0}^{\infty} u(C_t, E_{2t}) e^{-\rho t} dt \tag{6}$$

and

$$u = \theta \overline{C} min\{C, E_{2t}\} - 1_{\delta}\theta [min\{C, E_{2t}\}]^2 \tag{7}$$

where $E_{2t} = min\{E_{2st}, E_{2xt}\}$, $e^{-\rho t}$ is the discount factor, $\rho \neq 0$ and 1_{δ} is the indicative function that is equal to 0 during the period before the occurrence of the catastrophic event and 1 otherwise.

The first expression in equation (7) corresponds to the satisfaction the household gets from consuming the final goods and using energy services while the second part is the quadratic damage due to the catastrophic event. Households own firms in both energy and final goods sector. Part of the final good production is used as investment in its own sector, the second part in the energy production and the other part for consumption. Investments in productive sectors (final goods and energy goods) are capital accumulation and the market clearing conditions are the following:

$$Y_t = C_t + \dot{K}_t \tag{8}$$

$$\dot{K}_t = \dot{K}_{1t} + \dot{K}_{2t} \tag{9}$$

$$E_{st} + E_{xt} = E_1 + E_2 \tag{10}$$

2.4 Pollution threat

The only source of pollution in this model is from the NRE use by households and in the final goods sector. The NRE is mainly responsible for CO2 emission and pollutes environment. Pollution accumulates in environment with natural cleaning capacity α according to the following process:

$$\dot{Z}_t = E_{st} - \alpha Z \tag{11}$$

We suppose a pollution threat that corresponds to a stochastic critical pollution threshold $\overline{Z}$ above which households experience quadratic damages (following flooding for instance). The catastrophic event may occur at any state of pollution with a probability distribution function that we define as $F(Z) = Pr(Z > \overline{Z})$ and a density function $f(Z) = \frac{dF(Z)}{dQ}$. From this definition, we can deduce the hazard rate as $\Gamma(Z) = \frac{f(Z)}{1-F(Z)}$. The catastrophic event is more likely to happen in the case the economy faces an increasing pollution. Hence, this requires the hazard rate $\Gamma(Z)$ to be monotone non decreasing such that: $\Lambda^{'}(Z) \geq 0$

In the following sections, we analyze the optimal energy transition path under the stochastic pollution threshold $\overline{Z}$. Moreover, we determine the optimal level of pollution at the steady state that we compare to the deterministic case to deduce the possible effects of uncertainty.

3 Optimal energy transition path

In this section, we analyze the optimal energy transition and consider two types of regime switch: energy regime switch and uncertain pollution regime switch. Three regimes may occur that correspond to the energy transition path. In the first one, energy is produced by both oil and the renewable resource that are complementary and the pollution is below the threshold. At that time, the economy faces a risk of catastrophic events which occurrence is uncertain but it is more likely to happen in the case the economy faces an increasing pollution. After the catastrophic event has occurred, the economy switches to the second regime in which both energy sources are still used but it has already crossed the critical pollution threshold. During this second regime, the economy starts facing damages through a loss in utility. Only renewable resources are used in the third regime but the economy is still facing the negative consequences of the catastrophic event.

We backward solve the model by starting the resolution from the third regime (only RE) followed by the second regime and lastly the first regime. We use the optimality conditions as in Boucekkine et alii (2013) and the steady state in the first regime to find the optimal times to cross the pollution threshold and the full adoption of the RE and the steady state level of the pollution in the first regime. As it is not possible to get an analytical solution for the switching time, we numerically solve it.

3.1 Third regime

In the third regime, the economy uses only renewable energy and faces the negative consequences of the catastrophic event. In this case, the social planner maximizes the sum of discounted post event utility subject to the constraint of capital accumulation. During this third regime, the constraint of pollution accumulation is irrelevant as the economy has already crossed the critical pollution threshold and is facing its negative consequences. By using the equality in Leontief production function refereed as "Leontief condition" due to solutions being interior, the equation of capital accumulation (8) becomes $\dot{K}_t = K_{2t} - C_t$, with $Y = min\{K_2, E_{1t}\} = K_{2t} = E_{1t}$. Moreover, we know that the capital is split into final goods sector (K_{2t}) and energy sector (K_{1t}). The latter serves to produce the total energy (full renewable energy adoption) such that $K_{2t} = K_t - (E_{1t} + E_{2t})$. Then the equation of capital accumulation is : $\dot{K}_t = K_t - (E_{1t} + E_{2t}) - C_t$. The "Leontief condition" in the final goods sector implies that $E_{1t} = \frac{1}{2}(K_t - E_{2t})$ and that of the utility function implies that $C_t = E_{2t}$. Plugging the last two equalities in the equation of capital accumulation leads to:

$$\dot{K}_t = \frac{1}{2}K_t - \frac{3}{2}C_t. \tag{12}$$

The program to be solved by the social planner is the following:

$V_3 = Max \int_{T_2}^{\infty} (\theta C(\overline{C} - C))e^{-\rho(t-T_2)}dt$
st $\dot{K} = \frac{1}{2}K - \frac{3}{2}C$

The corresponding Hamiltonian is defined as:

$$H_3 = \theta C(\overline{C} - C) + \lambda(\frac{1}{2}K - \frac{3}{2}C)$$

with λ the co-state variable related to capital K.

The first order conditions (FOCs) are:

$$H_C = 0 \Longrightarrow \theta\overline{C} - 2\theta C = \frac{3}{2}\lambda \tag{13}$$

and

$$\frac{\dot{\lambda}}{\lambda} = \rho - \frac{1}{2} \tag{14}$$

The equation (13) highlights the traditional consumption versus savings arbitrage condition. It states that the marginal value of one more unit of invested capital has to be equalized to the marginal utility loss of foregone consumption such a saving decision implies. The second condition (14) defines the instantaneous return over capital as being constant over time.

(14) and (13) imply that:

$$\lambda_t = \lambda_{T_2} e^{(\rho-\frac{1}{2})(t-T_2)}$$

and

$$C_t = \frac{\overline{C}}{2} - \frac{3\lambda_{T_2}}{4\theta} e^{(\rho-\frac{1}{2})(t-T_2)}.$$

By using the expression of C_t in the equation (12) and the transversality conditions we can solve for capital K_t (see **Appendix A1**):

$$K_t = \frac{3\overline{C}}{2} + \frac{9\lambda_{T_2}}{8\theta(\rho-1)} e^{(\rho-\frac{1}{2})(t-T_2)} \tag{15}$$

Moreover, the expression of C_t in the value function gives (see **Appendix A2**):

$$V_3 = \frac{\theta\overline{C}^2}{4\rho} + \frac{\lambda_{T_2}^2}{16(\rho-1)}$$

At t=T_2, we have that:

$$K_{T_2} = \frac{3\overline{C}}{2} + \frac{9\lambda_{T_2}}{8\theta(\rho-1)}$$

Hence,

$$V_3 = \frac{\theta\overline{C}^2}{4\rho} + \frac{1}{16(\rho-1)}[(K_{T_2} - \frac{3\overline{C}}{2})\frac{8\theta(\rho-1)}{9}]^2.$$

3.2 Second regime

During the second regime, both energy sources are still used but the economy has already crossed the critical pollution threshold and starts facing damages. The social planner maximizes the sum of discounted post event utility and the discounted value function of the third regime subject to the constraint of capital accumulation and that of the NRE accumulation. The constraint of pollution accumulation is still irrelevant as the economy has already crossed the critical pollution threshold. The first difference here is the fact that the accumulation of the Non Renewable Energy source is relevant, as the economy is still using the oil after the catastrophic event has occurred. Also, during the second regime the economy uses a part of capital K_1 to only produce the Renewable energy as a part of the total energy capacity: $K_1 = E_{1xt} + E_{2xt}$. Because of the "Leontief conditions" in the energy uses $E_{1t} = min\{E_{1st}, E_{1xt}\} = E_{1st} = E_{1xt}$; $E_{2t} = min\{E_{2st}, E_{2xt}\} = E_{2st} = E_{2xt}$ and that of the utility function and of final goods production, we have the same equation of capital accumulation as in (12). For the same reasons, the dynamics of the NRE $dS_t = -(E_{1st} + E_{2st})dt = -(E_{1t} + E_{2t})dt$ becomes (see **Appendix A3**):

$$\frac{dS_t}{dt} = -(\frac{1}{2}K_t + \frac{1}{2}C_t) \tag{16}$$

The program to be solved by the social planner during the second regime is the following:

$$V_2 = Max \int_{T_1}^{T_2} (\theta C(\overline{C} - C))e^{-\rho(t-T_1)}dt + V_3 e^{-\rho T_2}$$

$$\text{st} \begin{cases} \dot{S} &= -(\frac{1}{2}K + \frac{1}{2}C) \\ \dot{K} &= \frac{1}{2}K - \frac{3}{2}C \end{cases}$$

The corresponding Hamiltonian can be written as:

$$H_2 = \theta C(\overline{C} - C) + \lambda_1(\frac{1}{2}K - \frac{3}{2}C) - \lambda_2(\frac{1}{2}K + \frac{1}{2}C).$$

with λ_1 and λ_2 the co-state variable associated to the capital K and the stock of the NRE S_t, respectively.

The FOCs are:

$$H_C = 0 \Rightarrow \theta\overline{C} - 2\theta C = \frac{3}{2}\lambda_1 + \frac{1}{2}\lambda_2 \tag{17}$$

$$\dot{\lambda_1} = (\rho - \frac{1}{2})\lambda_1 + \frac{1}{2}\lambda_2 \tag{18}$$

$$\frac{\dot{\lambda}_2}{\lambda_2} = \rho \tag{19}$$

The equation (17) is the usual arbitrage condition. It states that the marginal value of one more unit of consumption has to be balanced with the marginal value loss of foregone investment and the NRE depletion such a consumption decision implies. Contrary to the third regime, it appears in the arbitrage condition the value of the stock of the NRE due to the fact that it is limited in time. The instantaneous return over capital is no longer constant over time. The condition (18) states that the instantaneous return over capital depends on the relative value of the NRE stock with respect to that of the capital. This comes from the complementarity between capital and energy use in the production of final goods that expresses the level of energy use as a function of capital. Finally, the condition (19) defines that the growth of the value of the NRE is constant over time.

Equation (19) implies that:

$$\lambda_2 = \lambda_{2.T_1} e^{\rho(t-T_1)} \tag{20}$$

Using (20) in (18) helps to solve for λ_{1t} (see **Appendix A4**):

$$\lambda_{1t} = \lambda_{2.T_1} e^{\rho(t-T_1)} + (\lambda_{1.T_1} - \lambda_{2.T_1}) e^{(\rho-\frac{1}{2})(t-T_1)} \tag{21}$$

Also, using (20) and (21) in (17), we solve for C_t:

$$C_t = \frac{1}{2}\overline{C} - \frac{3}{4\theta}\lambda_{2.T_1} e^{\rho(t-T_1)} - \frac{3}{4\theta}(\lambda_{1.T_1} - \lambda_{2.T_1}) e^{(\rho-\frac{1}{2})(t-T_1)} - \frac{1}{4\theta}\lambda_{2.T_1} e^{\rho(t-T_1)}$$

Let use the expression of the consumption C_t in (12) to solve the equation of capital accumulation for K_t. We get that (see **Appendix A5**):

$$K_t = \frac{3}{2}\overline{C} + A_1(t)\lambda_{2.T_1} + A_2(t)\lambda_{1.T_1} + (K_{T_1} - \frac{3}{2}\overline{C}) e^{\frac{1}{2}(t-T_1)}$$

Where

$$A_1(t) = \frac{12}{8\theta(\rho-\frac{1}{2})} e^{\rho(t-T_1)} - \frac{9}{8\theta(\rho-1)} e^{(\rho-\frac{1}{2})(t-T_1)} + \frac{15-6\rho}{8\theta(\rho-1)(2\rho-1)} e^{\frac{1}{2}(t-T_1)}$$

and

$$A_2(t) = \frac{9}{8\theta(\rho-1)} e^{(\rho-\frac{1}{2})(t-T_1)} - \frac{9}{8\theta(\rho-1)} e^{\frac{1}{2}(t-T_1)}.$$

Now, let use the fact that the NRE is exhaustible and we have crossed the second regime after a period of time T_1.

$$S_0 = \underbrace{\int_0^{T_1} (E_{1t} + E_{2t})dt}_{\overline{Z}} + \int_{T_1}^{T_2} (E_{1t} + E_{2t})dt$$

This implies that:

$$S_0 - \overline{Z} = \frac{1}{2}\int_{T_1}^{T_2} (K_t + C_t)dt$$

with $S_0 > \overline{Z}$ [4].

By replacing K_t and C_t with their corresponding expressions, we get the following equation(see **Appendix A6**):

$$W_1(\Delta)\lambda_{2.T_1} + W_2(\Delta)\lambda_{1.T_1} = 2(S_0 - \overline{Z}) - 2\overline{C}\Delta - 2K_{T_1}(e^{\frac{1}{2}\Delta} - 1) + 3\overline{C}(e^{\frac{1}{2}\Delta} - 1) \quad (22)$$

Where

$$\Delta = T_2 - T_1$$

$$W_1(\Delta) = \frac{-\rho+2}{\theta\rho(\rho-\frac{1}{2})}e^{\rho\Delta} + \frac{6\rho-15}{8\theta(\rho-1)(\rho-\frac{1}{2})}e^{(\rho-\frac{1}{2})\Delta} + \frac{15-6\rho}{4\theta(\rho-1)(2\rho-1)}e^{\frac{1}{2}\Delta} + \frac{2\rho^2-9\rho+16}{8\theta\rho(\rho-1)(\rho-\frac{1}{2})}$$

and

$$W_2(\Delta) = \frac{-6\rho+15}{8\theta(\rho-1)(\rho-\frac{1}{2})}e^{(\rho-\frac{1}{2})\Delta} - \frac{9}{4\theta(\rho-1)}e^{\frac{1}{2}\Delta} + \frac{3}{\theta(\rho-\frac{1}{2})}.$$

The equation (22) expresses the relationship between the value of the capital ($\lambda_{1.T_1}$) and that of the NRE stock ($\lambda_{2.T_1}$) at the beginning of the second regime (T_1) according to the constraint of the NRE accumulation. In order to determine the value of $\lambda_{1.T_1}$ and that of $\lambda_{2.T_1}$, we need a second relationship that may come from the following optimality conditions.

3.3 Optimality conditions

Following Boucekkine et alii (2013), we use three types of boundary conditions: Continuity of λ_1 [5], Continuity of K and the equality of the Hamiltonian at the switching time t= T_2. Here, we do not use the optimality conditions at t= T_1 because the switching time to the second regime is uncertain.

First, the continuity of λ_1 at t= T_2 implies that:

$$\lambda_{T_2} = \lambda_{1.T_2} = \lambda_{2.T_1}e^{\rho\Delta} + (\lambda_{1.T_1} - \lambda_{2.T_1})e^{(\rho-\frac{1}{2})\Delta}$$

Then, we can rewrite the value of the capital at the beginning of the third regime (T_2) as:

$$K_{T_2} = \frac{3}{2}\overline{C} + B_1(\Delta)\lambda_{2.T_1} + B_2(\Delta)\lambda_{1.T_1} \quad (23)$$

Where

$$B_1(\Delta) = \frac{9}{8\theta(\rho-1)}e^{\rho\Delta}(1 - e^{-\frac{1}{2}\Delta})$$

[4] This is a necessary condition to have relevant pollution threat, otherwise the economy could completely deplete the NRE without crossing the pollution threshold.

[5] The co-state variable λ_2 associated with the pollution stock Z is not continuous at the switching time T_1 because Z is fixed to $\overline{Z}$. At the switching time T_2, it can be freely chosen and becomes continuous but it no longer exist during the third regime because the RE is not polluting.

and

$$B_2(\Delta) = -\frac{9}{8\theta(\rho-1)}e^{(\rho-\frac{1}{2})\Delta}.$$

Taking the expression of the capital K_t during the second regime at t= T_2, we have:

$$K_{T_2} = \frac{3}{2}\overline{C} + A_1(T_2)\lambda_{2.T_1} + A_2(T_2)\lambda_{1.T_1} + (K_{T_1} - \frac{3}{2}\overline{C})e^{\frac{1}{2}\Delta}. \tag{24}$$

The continuity of capital at t= T_2 implies that (24)=(23).
Hence,

$$Z_1\lambda_{2.T_1} + Z_2\lambda_{1.T_1} = K_{T_1}e^{\frac{1}{2}\Delta} - \frac{3}{2}\overline{C}e^{\frac{1}{2}\Delta} \tag{25}$$

Where, $Z_1 = B_1 - A_1$ and $Z_2 = B_2 - A_2$

The equation (25) also expresses the relationship between the value of the capital ($\lambda_{1.T_1}$) and that of the NRE stock ($\lambda_{2.T_1}$) at the beginning of the second regime (T_1) according to the optimality conditions.

To determine $\lambda_{1.T_1}$ and $\lambda_{2.T_1}$, one just need to solve simultaneously the equations (22) and (25) (see **Appendix A7**): $\lambda_{1.T_1}$ and $\lambda_{2.T_1}$ are linear in K_{T_1}. From that, we can deduce the expression of the value function in the second regime as:

$$V_2 = f(K_{T_1}, \Delta),$$

where f is a polynomial of second degree in K_{T_1}.

3.4 First energy regime (NR, $\underline{Z}$)

The economy starts using both energy sources and the pollution level is under the critical pollution threshold. During this first period, the economy faces a risk of catastrophic events which occurrence is uncertain but it is more likely to happen in the case the economy faces an increasing pollution. The social planner maximizes the expected sum of discounted pre-event utility and the discounted value function of the second regime subject to the constraint of capital accumulation and that of the pollution accumulation. The constraint of pollution accumulation is relevant because of the risk to cross the critical threshold. Also, as the NRE is abundant, we do not need to consider the accumulation of the NRE because the economy will cross the pollution threshold before the complete depletion of the NRE. The equation of the NRE accumulation can be expressed as a function of that of the pollution: $dZ_t = dS_t - \alpha Z dt$. We analyze the deterministic case as a benchmark and the stochastic pollution threshold.

3.4.1 Deterministic case

The program to be solved by the social planner in the deterministic case is the following:

$V_1^d = Max \int_0^{T_1} \theta C\overline{C}e^{-\rho t}dt + V_2 e^{-\rho T_1}$

$$\text{st}\begin{cases}\dot{K} &= \frac{1}{2}K - \frac{3}{2}C\\ \dot{Z} &= \frac{1}{2}K + \frac{1}{2}C - \alpha Z,\ S_0 > \overline{Z}\end{cases}$$

The Hamiltonian can be written as:

$$H_1^d = \theta C\overline{C} + \lambda_1(\frac{1}{2}K - \frac{3}{2}C) + \lambda_2(\frac{1}{2}K + \frac{1}{2}C - \alpha Z).$$

FOCs are given by:

$$H_C^d = 0 \Rightarrow \lambda_2 = -2\theta\overline{C} + 3\lambda_1 \tag{26}$$

$$\dot{\lambda_1} = \rho\lambda_1 - \frac{\lambda_1}{2} - \frac{\lambda_2}{2} \tag{27}$$

$$\dot{\lambda_2} = \rho\lambda_2 + \alpha\lambda_2 \tag{28}$$

First, remark here that it is not possible to have a steady state. From (28), $\dot{\lambda_2} = 0$ implies that $\rho + \alpha$ =0 which is impossible ($\rho > 0$ and $\alpha > 0$). Moreover, neither the state variables, nor the control variables appear in the FOCs. The FOCs help to determine only the co-state variables. This implies that the level of the control variables can be freely chosen. Thus, the maximum Principle applied to our model fails to yield the optimal transition path from the first to the second regime under certainty pollution threshold.

3.4.2 Stochastic case

In the stochastic case, the program to be solved by the social planner is the following:

$$V_1 = Max \int_0^\infty [\theta C\overline{C}(1 - F_\tau(t)) + f_\tau(t)V_2(K_t, \Delta)e^{-\rho t}]dt$$

$$\text{st}\begin{cases}\dot{K} &= \frac{1}{2}K - \frac{3}{2}C\\ \dot{Z} &= \frac{1}{2}K + \frac{1}{2}C - \alpha Z,\ S_0 > \overline{Z}\end{cases}$$

First, let us focus on the probability to remain in the first regime (under the pollution threshold) $1 - F_\tau(t) = Pr(t < \tau)$ and the probability to irreversibly cross the pollution threshold $f_\tau(t) = \frac{dF_\tau(t)}{dt}$, where τ is the switching time to the second regime. The distribution of τ depends on that of the pollution so that one can derive the distribution of τ from the trajectories of the pollution. As in Ayong et al. (2011), we consider monotone trajectories for the pollution. From that, we can define the distribution of τ as:

$$1 - F_\tau(t) = Pr(t < \tau/\tau > 0) = Pr(Z(t) < \underline{Z}/Z_0 < \underline{Z}) = \frac{1 - F(Z(t))}{F(Z_0)}$$

and

$$f_\tau(t) = \frac{f(Z)\dot{Z}}{F(Z_0)} = \frac{f(Z)(\frac{1}{2}K + \frac{1}{2}C - \alpha Z)}{F(Z_0)}$$

The Hamiltonian can be written as:

$$H_1^s = \theta C\overline{C}(1 - F(Z)) + f(Z)\dot{Z}V_2(K_t, \Delta) + \lambda_1(\frac{1}{2}K - \frac{3}{2}C) + \lambda_2(\frac{1}{2}K + \frac{1}{2}C - \alpha Z).$$

Before deriving the FOCs, let assume the uniform distribution: $F(Z) = a(Z - Z_0)$, $f(Z) = a$, with Z_0=0. This implies that the hazard rate is $\Lambda(Z) = \frac{a}{1-aZ}$ and $\Lambda^{'} > 0$.

FOCs are given by:

$$H_C^s = 0 \Rightarrow \lambda_2 = -2\theta\overline{C}(1 - aZ) + 3\lambda_1 \tag{29}$$

$$\dot{\lambda_1} = \rho\lambda_1 - a\dot{Z}\frac{\partial V_2}{\partial K} - \frac{a}{2}V_2 - \frac{\lambda_1}{2} - \frac{\lambda_2}{2} \tag{30}$$

$$\dot{\lambda_2} = \rho\lambda_2 + \theta a\overline{C}C + a\alpha V_2 + \alpha\lambda_2 \tag{31}$$

Using (29) , (30) , (31) and the steady state $\dot{\lambda_1} = \dot{\lambda_2} = \dot{K} = \dot{Z} = \dot{C} = 0$, we get that:

$$C = \frac{(\rho+\alpha)(2\rho-1)}{a(\rho-2)} - \frac{(\rho+\alpha)(2\rho-1)}{\rho-2}Z - \frac{3(\rho+\alpha)}{2(\rho-2)\theta\overline{C}}V_2(K) - \frac{\alpha}{\theta\overline{C}}V_2(K) = \frac{\alpha}{2}Z$$

with $K = \frac{3\alpha}{2}Z$.

The Focs show that the consumption is a decreasing (resp. increasing) function of the pollution level as long as the discount factor is less (resp. higher) than a half, while the steady state of the pollution dynamics leads to an increasing function of the level of pollution. Solving the above equation, we can find the steady state level of the pollution Z^* and derive that of the capital, the consumption and the energy use. As it is not possible to get an analytical solution, we numerically solve for Z^*.

4 Numerical results and sensitivity analysis

The issue here is to show that there is numerical value for the parameters of our model that solves for the steady state level of the pollution. After that numerical solution, we perform the sensitivity analysis to identify the possible effect of those parameters and the uncertainty on the steady state level of the pollution.

4.1 Numerical results

The parameters values are chosen as follows. As we are concerned about environmental issues (pollution) that can lead to catastrophic event, we set the discount rate ρ to 0.05 so that people are more patient and concerned about the consequences of their behaviour in the future. The sensitivity analysis will help to point out how the pollution would evolve in the case people are impatient (high discount factor). There is no evidence in economics and scientific literature about the cleaning capacity of the environment. Thus, we assume a natural regeneration rate of the environment α equals to 0.3 that is more optimistic than that of 0.1 in Tahvonen (1997). The parameters of the utility function θ and $\overline{C}$ are set to 0.07 and 9, respectively. We consider an initial stock of the NRE S_0 equals to 28000 and the pollution threshold $\overline{Z} = 1000$ as benchmark. The uncertainty parameter a is equal to 0.5 without initial pollution ($Z_0 = 0$).

From the above parameters values, numerical resolution show that the steady sate level of pollution $Z^*= 801.84$. This steady level of pollution can be compared to the benchmark threshold $\overline{Z}$=1000. One can conclude that it exists numerical values that correspond to a lower pollution level. The introduction of uncertainty may reduce the pollution level. The fact that people are uncertain about the occurrence of the catastrophic event and know that this may probably happen in the case they are more polluting, may refrain them from polluting. The sensitivity analysis on the uncertainty parameter will help to understand the real impact of the uncertainty on the pollution.

4.2 Sensitivity analysis

The discount rate ρ negatively affect the level of pollution. At first glance, this result is a bit surprising in the sense that more impatient people should consume and pollute more because they do not care about the consequences of the pollution in the future. However, the fact that the RE and NRE uses are complementary and a part of capital is used to produce energy can justify the controversial effect. In fact, people are more impatient so that they could consume more leading to less investment, especially in renewable energy. As the RE is complementary with the NRE, the latter is less used and the economy is less polluting. Moreover, as the damage is highly negatively sensitive to consumption, people may refrain from consuming more in order to avoid high damage. This may also lead to less level of pollution. The cleaning process is beneficial for the economy as it is costless and helps to reduce the level of pollution. The level of the pollution at the steady state negatively depends on the natural regeneration rate of the environment α. The more the environment is capable to clean the pollution, the less the pollution remains in the environment.

As the coefficient of the damage θ is low, one should expect an increase in the level of pollution. But, as the damage negatively depends on the consumption that positively depends on the level of pollution, the economy is less consuming and less polluting. The steady level of pollution is low when the benchmark level of pollution is low. People seems to less pollute as soon as they know that the critical pollution threshold $\overline{Z}$ is low. The availability of the polluting resources positively affects the level of pollution in the sense that, the more the NRE is available, the more the economy is willing to consume and pollute. Finally, the steady state level of pollution is negatively sensitive to the uncertainty parameter a. The less the uncertainty parameter a is, the low is the probability of the occurrence of the catastrophic event that induces the switch to the second regime. As the risk of the catastrophic event is low, the economy may not care much about the problem of pollution. In this case it may be optimal for the economy to extract and use more the polluting resources and then to pollute more. Hence, the economy could end up with high steady level of the pollution.

5 Certainty versus Uncertainty pollution threshold

There are many differences in the results when we consider the uncertainty on the pollution threshold compared to the certainty case. First, the steady state is not possible in the deterministic case. This means that the economy could not constantly consume the final goods and the energy and invest without uncertainty. Moreover, as neither the state variables, nor the control variables appear in the FOCs, the level of the control variables can be freely chosen. One can then conclude that, compared to the stochastic case, the deterministic case does not yield the optimal transition path from the first to the second regime under certainty pollution threshold. Secondly, the level of pollution can be affected by the uncertainty. According to the numercial results, the introduction of uncertainty may negatively affect the pollution level. By introducing uncertain on the occurrence of the catastrophic event and knowing that this may probably happen in the case they are more polluting, people may have incentives to less use the polluting resources. The sensitivity analysis on the uncertainty parameter helps to confirm and better understand the real impact of the uncertainty on the pollution. As the uncertainty decreases (resp. increases) so that the probability of the occurrence of the catastrophic event is low (resp. high), the economy may not care much (resp. less) about the problem of pollution. For low uncertainty, the optimal solution may result in high use of the polluting resources and then more pollution. One can conclude that uncertainty may favour the energy transition in the sense that people may fear about what will happen in the future. As they are not sure about the occurrence of the climate change impacts, they may take some precautions of prevention that could give incentives to reduce the pollution. Those incentives may help to adopt the backstop technology as an alternative or/and invest in energy savings technology. However, as long as the uncertainty is high, they may exist some people who will be skeptical and may prefer not to make decisions for the purpose of precaution.

6 Conclusion

In the present paper, we analyze the optimal energy transition of a two-sector economy with exhaustible oil reserves, a renewable source of energy and a stochastic critical pollution threshold above which a catastrophic event (following flooding for instance) may occur. Three regimes may occur that correspond to the optimal energy transition path. In the first one, energy is produced by both oil and the renewable resource that are complementary and the pollution is below the threshold. At that time, the economy faces a risk of catastrophic events which occurrence is uncertain but it is more likely to happen in the case the economy faces an increasing pollution. After the catastrophic event has occurred, the economy switches to the second regime in which both energy sources are still used but it has already crossed the critical pollution threshold. During this second regime, the economy starts facing quadratic damages through a loss in utility. Only renewable resources are used in the third regime but the economy is still facing the negative consequences of the catastrophic event. We backward solve the model and use the optimality conditions as in Boucekkine et alii (2013) and the steady state in the first regime to find the optimal times to cross the pollution threshold and the full adoption of the RE and the steady state level of the pollution.

Numerical results show that there exist numerical values that lead to a pollution level lower than that of the deterministic threshold. The introduction of uncertainty in our model may have two main effects. First, compared to the stochastic case, the deterministic case does not yield the optimal transition path from the first to the second regime under certainty pollution threshold. As neither the state variables, nor the control variables appear in the FOCs, the level of the control variables can be freely chosen. Secondly, the level of pollution may be affected by the uncertainty. Because of uncertainty, people may have incentives to less use the polluting resources. Moreover, for low uncertainty, the optimal solution may result in high use of the polluting resources and then more pollution. One can conclude that uncertainty may favour the energy transition in the sense that people may fear about the future. As they are not sure about the occurrence of the climate change impacts, they may take some precautions of prevention that could give incentives to reduce the pollution. Those incentives may help to adopt the backstop technology as an alternative or/and invest in energy savings technology. However, as long as the uncertainty is high, they may exist some people who will be skeptical and may prefer not to make decisions for the precautionary purpose.

7 References

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8 Appendix

8.1 Appendix A1

To determine the expression of capital in the third regime, we need to replace the expression of the consumption given by the FOCs into the equation of capital accumulation (12) and use the transversality conditions and solve for the capital K_t. The expression of the consumption $C_t = \frac{\overline{C}}{2} - \frac{3\lambda_{T_2}}{4\theta} e^{(\rho-\frac{1}{2})(t-T_2)}$ in (12) gives:

$$\dot{K}_t - \frac{1}{2}K_t = -3\frac{\overline{C}}{4} + \frac{9\lambda_{T_2}}{8\theta} e^{(\rho-\frac{1}{2})(t-T_2)}.$$

Let us make a change of variables as follows: $x = Ke^{-\frac{1}{2}(t-T_2)}$. This implies that:

$$\dot{x}e^{\frac{1}{2}(t-T_2)} = -3\frac{\overline{C}}{4} + \frac{9\lambda_{T_2}}{8\theta} e^{(\rho-\frac{1}{2})(t-T_2)}$$

$$\Rightarrow \dot{x} = -3\frac{\overline{C}}{4} e^{-\frac{1}{2}(t-T_2)} + \frac{9\lambda_{T_2}}{8\theta} e^{(\rho-1)(t-T_2)}.$$

The solution for the above differential equation in x is:

$$x_t = 3\frac{\overline{C}}{2} e^{-\frac{1}{2}(t-T_2)} + \frac{9\lambda_{T_2}}{8\theta(\rho-1)} e^{(\rho-1)(t-T_2)} + \overline{x}$$

The solution for the capital can be derived as:

$$K_t = \frac{3\overline{C}}{2} + \frac{9\lambda_{T_2}}{8\theta(\rho-1)} e^{(\rho-\frac{1}{2})(t-T_2)} + \overline{x}e^{\frac{1}{2}(t-T_2)}$$

Now, let us use the transversality conditions to determine the value of $\overline{x}$:

$$\lim_{t\to\infty} K_t \lambda_t e^{-\rho(t-T_2)} = \lim_{t\to\infty} K_t \lambda_{T_2} e^{-\frac{1}{2}(t-T_2)} = 0$$

$$\Rightarrow \lim_{t\to\infty} \frac{3\overline{C}}{2} \lambda_{T_2} e^{-\frac{1}{2}(t-T_2)} + \frac{9}{8\theta(\rho-1)} \lambda_{T_2}^2 e^{(\rho-1)(t-T_2)} + \overline{x}\lambda_{T_2} = 0$$

$\Leftrightarrow \overline{x} = 0$ with $\rho \leq 1$ and $\lambda_{T_2} \neq 0$.

Hence, the expression of the capital can be written as:

$$K_t = \frac{3\overline{C}}{2} + \frac{9\lambda_{T_2}}{8\theta(\rho-1)} e^{(\rho-\frac{1}{2})(t-T_2)}$$

8.2 Appendix A2

To determine the value function of the third regime, we just need to replace the consumption into the value function and solve it.

Let us recall that:

$$V_3 = \int_{T_2}^{\infty} (\theta C(\overline{C} - C)) e^{-\rho(t-T_2)} dt$$

The expression of C_t in the above expression gives:

$$V_3 = \int_{T_2}^{\infty} [\frac{\theta \overline{C}^2}{4} e^{-\rho(t-T_2)} - \frac{\lambda_{T_2}^2}{16\theta} e^{(\rho-1)(t-T_2)}] dt$$

$$\Rightarrow V_3 = -\frac{\theta \overline{C}^2}{4\rho} [e^{-\rho(t-T_2)}]_{T_2}^{\infty} - \frac{\lambda_{T_2}^2}{16\theta(\rho-1)} [e^{(\rho-1)(t-T_2)}]_{T_2}^{\infty}.$$

This leads to:

$$V_3 = \frac{\theta \overline{C}^2}{4\rho} + \frac{\lambda_{T_2}^2}{16(\rho-1)}$$

8.3 Appendix A3

The issue here is to express the dynamics of the NRE as a function of the capital and the consumption. Because of the "Leontief conditions" in the production of final goods, we have that: Y=$K_2 = E_1$. Also, the "Leontief conditions" in the energy uses imply that $E_{1t} = E_{1xt} = E_{1st}$ and $E_{2t} = E_{2xt} = E_{2st}$ and the capital is invested to produce final goods and the RE: $K_t = K_{1t} + K_{2t} = K_{1t} + E_{2xt} + E_{1xt}$. Then, we can deduce that $Y_t = K_{2t} = E_{1t} = K_t - E_{2t} - E_{1t}$, that implies that $E_{1t} = \frac{1}{2}(K_t - E_{2t})$.

By summing up E_{1t} and E_{2t}, we get that:

$$E_{1t} + E_{2t} = \frac{1}{2}(K_t - E_{2t}) + E_{2t} = \frac{1}{2}(K_t + E_{2t})$$

Finally, the "Leontief conditions" in the utility function gives $C_t = E_{2t}$ that we replace in the above equation to find that:

$$E_{1t} + E_{2t} = \frac{1}{2}(K_t + C_t)$$

Hence, the dynamics of the NRE $dS_t = -(E_{1st} + E_{2st})dt = -(E_{1t} + E_{2t})dt$ becomes:

$$\frac{dS_t}{dt} = -(\frac{1}{2}K_t + \frac{1}{2}C_t)$$

8.4 Appendix A4

The value of the capital λ_{1t} during the second regime can be determined with the FOCs (20) and (18) (in the text). We have to solve the following differential equation for λ_{1t}:

$$\dot{\lambda_{1t}} - (\rho - \frac{1}{2})\lambda_{1t} = \frac{\lambda_{2.T_1}}{2} e^{\rho(t-T_1)}.$$

By making a change of variables $x = \lambda_{1t} e^{-(\rho - \frac{1}{2})(t-T_1)}$, the above differential equation becomes:

$$\dot{x_t} = \frac{\lambda_{2.T_1}}{2} e^{\frac{1}{2}(t-T_1)}.$$

One can derive the solution as :

$$\lambda_{1t} = \lambda_{2.T_1} e^{\rho(t-T_1)} + \overline{x} e^{(\rho - \frac{1}{2})(t-T_1)}$$

To determine the unknown $\overline{x}$, we take the above expression at t=T_1:

$$\overline{x} = \lambda_{1.T_1} - \lambda_{2.T_1}$$

Hence,

$$\lambda_{1t} = \lambda_{2.T_1} e^{\rho(t-T_1)} + (\lambda_{1.T_1} - \lambda_{2.T_1}) e^{(\rho - \frac{1}{2})(t-T_1)}$$

8.5 Appendix A5

Let use the expression of the consumption C_t in (12) to solve the equation of capital accumulation for K_t. The equation of capital accumulation (12) becomes:

$$\dot{K_t} - \frac{1}{2}K_t = -\frac{3}{2}[\frac{1}{2}\overline{C} - \frac{3}{4\theta}\lambda_{2.T_1} e^{\rho(t-T_1)} - \frac{3}{4\theta}(\lambda_{1.T_1} - \lambda_{2.T_1}) e^{(\rho - \frac{1}{2})(t-T_1)} - \frac{1}{4\theta}\lambda_{2.T_1} e^{\rho(t-T_1)}]$$

By making a change of variables as in the **Appendix A1** and by taking the level of the capital at t=T_1 (K_{T_1}) to determine the unknown $\overline{x}$, one get that:

$$K_t = \frac{3}{2}\overline{C} + A_1(t)\lambda_{2.T_1} + A_2(t)\lambda_{1.T_1} + (K_{T_1} - \frac{3}{2}\overline{C}) e^{\frac{1}{2}(t-T_1)}$$

Where

$$A_1(t) = \frac{12}{8\theta(\rho - \frac{1}{2})} e^{\rho(t-T_1)} - \frac{9}{8\theta(\rho - 1)} e^{(\rho - \frac{1}{2})(t-T_1)} + \frac{15 - 6\rho}{8\theta(\rho - 1)(2\rho - 1)} e^{\frac{1}{2}(t-T_1)}$$

and

$$A_2(t) = \frac{9}{8\theta(\rho - 1)} e^{(\rho - \frac{1}{2})(t-T_1)} - \frac{9}{8\theta(\rho - 1)} e^{\frac{1}{2}(t-T_1)}.$$

8.6 Appendix A6

Here, we use the constraint of the NRE accumulation to get a first relationship between the value of the capital ($\lambda_{1.T_1}$) and that of the NRE stock ($\lambda_{2.T_1}$) at the beginning of the second regime (T_1). To do that, let us assume that the NRE is exhaustible and we have crossed the second regime after a period of time T_1. Then, the initial stock of the NRE S_0 is equal to the sum of the part of the NRE that is used during the first regime and that corresponds to the total amount of pollution $\overline{Z}$ and the part of the NRE that the economy uses during the second regime. We have that:

$$S_0 - \overline{Z} = \int_{T_1}^{T_2} (E_{1t} + E_{2t})dt = \frac{1}{2}\int_{T_1}^{T_2} K_t dt + \frac{1}{2}\int_{T_1}^{T_2} C_t dt$$

with $S_0 > \overline{Z}$.

Let solve separately each part of the the Right Hand Side (RHS) of the above equation:

$$\chi_1 = \frac{1}{2}\int_{T_1}^{T_2} K_t dt$$

and

$$\chi_2 = \frac{1}{2}\int_{T_1}^{T_2} C_t dt.$$

We get that:

$$\chi_1 = \frac{1}{2}\int_{T_1}^{T_2} [\frac{3}{2}\overline{C} + A_1(t)\lambda_{2.T_1} + A_2(t)\lambda_{1.T_1} + (K_{T_1} - \frac{3}{2}\overline{C})e^{\frac{1}{2}(t-T_1)}]dt$$

$$\Rightarrow \chi_1 = \frac{1}{2}[D_0(\Delta) + D_1(\Delta)\lambda_{2.T_1} + D_2(\Delta)\lambda_{1.T_1}]$$

and

$$\chi_2 = \frac{1}{2}\int_{T_1}^{T_2} [\frac{1}{2}\overline{C} - \frac{3}{4\theta}\lambda_{2.T_1}e^{\rho(t-T_1)} - \frac{3}{4\theta}(\lambda_{1.T_1} - \lambda_{2.T_1})e^{(\rho-\frac{1}{2})(t-T_1)} - \frac{1}{4\theta}\lambda_{2.T_1}e^{\rho(t-T_1)}]dt$$

$$\Rightarrow \chi_2 = \frac{1}{2}[C_0(\Delta) + C_1(\Delta)\lambda_{2.T_1} + C_2(\Delta)\lambda_{1.T_1}]$$

Where

$$\Delta = T_2 - T_1,$$

$$D_0(\Delta) = \frac{3\overline{C}\Delta}{2} + (2K_{T_1} - 3\overline{C})(e^{\frac{\Delta}{2}} - 1),$$

$$C_0(\Delta) = \frac{\overline{C}\Delta}{2},$$

$$D_1(\Delta)=\frac{12}{8\theta\rho(\rho-\frac{1}{2})}(e^{\rho\Delta}-1)-\frac{9}{8\theta(\rho-1)(\rho-\frac{1}{2})}(e^{(\rho-\frac{1}{2})\Delta}-1)+\frac{15-6\rho}{4\theta(\rho-1)(2\rho-1)}(e^{\frac{1}{2}\Delta}-1),$$

$$C_1(\Delta)=-\frac{e^{\rho\Delta}}{\theta\rho}+\frac{1}{\theta\rho}+\frac{3}{4\theta(\rho-\frac{1}{2})}e^{(\rho-\frac{1}{2})\Delta}-\frac{3}{4\theta(\rho-\frac{1}{2})},$$

$$D_2(\Delta)=\frac{9}{8\theta(\rho-1)(\rho-\frac{1}{2})}(e^{(\rho-\frac{1}{2})\Delta}-1)-\frac{9}{4\theta(\rho-1)}(e^{\frac{1}{2}\Delta}-1),$$

and

$$C_2(\Delta)=-\frac{3}{4\theta(\rho-\frac{1}{2})}(e^{(\rho-\frac{1}{2})\Delta}-1)$$

Using the above expressions, we get that:

$$S_0-\overline{Z}=\chi_1+\chi_2=\frac{1}{2}[D_0+C_0+(D_1+C_1)\lambda_{2.T_1}+(D_2+C_2)\lambda_{1.T_1}]$$

Hence,

$$W_1(\Delta)\lambda_{2.T_1}+W_2(\Delta)\lambda_{1.T_1}=2(S_0-\overline{Z})-2\overline{C}\Delta-2K_{T_1}(e^{\frac{1}{2}\Delta}-1)+3\overline{C}(e^{\frac{1}{2}\Delta}-1)$$

Where

$$W_1(\Delta)=D_1+C_1$$

and

$$W_2(\Delta)=D_2+C_2.$$

8.7 Appendix A7

To determine $\lambda_{1.T_1}$ and $\lambda_{2.T_1}$, one just need to solve simultaneously the equations (22) and (25) (in the text):

$$\begin{cases} W_1\lambda_{2.T_1}+W_2\lambda_{1.T_1} &= 2(S_0-\overline{Z})-2\overline{C}\Delta-2K_{T_1}(e^{\frac{1}{2}\Delta}-1)+3\overline{C}(e^{\frac{1}{2}\Delta}-1)\\ Z_1\lambda_{2.T_1}+Z_2\lambda_{1.T_1} &= K_{T_1}e^{\frac{1}{2}\Delta}-\frac{3}{2}\overline{C}e^{\frac{1}{2}\Delta}\end{cases}$$

Using the substitution method, the solution of the above system of two linear equations in $\lambda_{1.T_1}$ and $\lambda_{2.T_1}$ is:

$$\lambda^*_{1.T_1}=\frac{1}{W_2Z_1-W_1Z_2}[2(S_0-\overline{Z})Z_1-2\overline{C}\Delta Z_1-2K_{T_1}(e^{\frac{1}{2}\Delta}-1)Z_1+3\overline{C}(e^{\frac{1}{2}\Delta}-1)Z_1-Z_3W_1K_{T_1}+\frac{3}{2}\overline{C}Z_3W_1]$$

and

$$\lambda^*_{2.T_1}=\frac{1}{Z_1}[Z_3K_{T_1}-\frac{3}{2}\overline{C}Z_3]-\frac{Z_2}{Z_1}\lambda^*_{1.T_1}$$

$\lambda^*_{2.T_1}$ and $\lambda^*_{1.T_1}$ are linear in K_{T_1}.

From that, we can deduce the expression of the value function in the second regime V_2.

Let us recall that:

$$V_2 = Max \int_{T_1}^{T_2} (\theta C(\overline{C} - C))e^{-\rho(t-T_1)}dt + V_3 e^{-\rho T_2}$$

Where

$$C_t = \frac{1}{2}\overline{C} - \frac{3}{4\theta}\lambda_{2.T_1}e^{\rho(t-T_1)} - \frac{3}{4\theta}(\lambda_{1.T_1} - \lambda_{2.T_1})e^{(\rho-\frac{1}{2})(t-T_1)} - \frac{1}{4\theta}\lambda_{2.T_1}e^{\rho(t-T_1)}$$

and

$$V_3 = \frac{\theta\overline{C}^2}{4\rho} + \frac{1}{16(\rho-1)}[(K_{T_2} - \frac{3\overline{C}}{2})\frac{8\theta(\rho-1)}{9}]^2.$$

One can easily solve V_2 and get that:

$$V_2 = \frac{\theta\overline{C}^2}{4\rho}(1-e^{-\rho\Delta}) + \frac{9}{16\theta}\frac{(\lambda_{1.T_1} - \lambda_{2.T_1})^2}{2(\rho-\frac{1}{2})}(1-e^{2(\rho-\frac{1}{2})\Delta}) + \frac{\lambda_{2.T_1}(\lambda_{1.T_1} - \lambda_{2.T_1})}{4\theta(2\rho-\frac{1}{2})}(1-e^{(2\rho-\frac{1}{2})\Delta}) + V_3 e^{-\rho T_2}$$

As $\lambda^*_{2.T_1}$ and $\lambda^*_{1.T_1}$ are linear in K_{T_1}, we have that:

$$V_2 = f(K_{T_1})$$

where f is a polynomial of second degree in K_{T_1}.