Environmental Policy and Growth in a Model with Endogenous Environmental Awareness*

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Abstract

This paper examines the relationship between environmental policy and growth when green preferences are endogenously determined by education and pollution. The government can implement a tax on pollution and recycle the revenue in public pollution abatement and/or education subsidy (influencing green behaviors). When agent's preferences for the environment are highly sensitive to environmental damages, the economy can converge to a balanced growth path equilibrium with damped oscillations. Therefore, we identify two objectives that environmental policy seeks to address: remove oscillations, source of intergenerational inequalities, and enhance the long-term growth rate. We show that a tighter tax allows to achieve both objectives when the tax revenue is well allocated between education and direct environmental protection.

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1 Introduction

The link between growth and the environment is a fundamental issue in environmental economics, as highlighted in the literature reviews of Brock and Taylor (2005) and Xepapadeas (2005). One of the main questions raised is the role of environmental policy in attaining a sustainable development, where economic growth is compatible with a non-damaging environment. To achieve such a goal, policy makers have several economic levers and can combine them, as underlined by OECD (2007).[1] The most obvious instrument is pollution taxation, introduced to reduce environmentally harmful

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[1]OECD (2007) highlights two main reasons to justify the use of instrument mixes: the "multi-aspect" nature of environmental issues and the fact that instruments can reinforce each other.

activities. But the governments may also invest in pollution abatement activities (e.g. water treatment, waste management, investment in renewable energy, conservation of forests...) or in more indirect interventions which aim to modify households' behaviors. For example, OECD (2008) refers to education as "one of the most powerful tools for providing individuals with the appropriate skills and competencies to become sustainable consumers". The importance of this instrument is also illustrated by the United Nations which declares the decade 2005-2014 as the "UN Decade of Education for Sustainable Development".[2]

In the light of the development of policies directed to consumers' behaviors, it seems particularly relevant to consider the role of agents' green preferences on environmental issues. Moreover, as European Commission (2008) points out, households are becoming more aware of environmental issues and of their role in environmental protection since the recent decades, reflecting that these preferences evolve over time. The purpose of this paper is thus to study how environmental policy affects growth performance, when individual preferences for the environment are endogenous.

A number of studies deals with the link between environmental policy and growth, however no consensus exists. Ono (2003a) underlines the intergenerational effect of environmental taxation. In his paper, tax reduces the polluting production, but improves the level of environmental quality bequeathed to future generation, such that there exists an intermediary level of tax that enhances the long-term growth. Other contributions examine this issue by considering that human capital accumulation is the engine of growth. In a Lucas model, Gradus and Smulders (1993) emphasize that an improvement in environmental quality has a positive effect on long-term growth when pollution affects directly human capital accumulation. More recent papers underline that, even without such assumption, a tighter environmental tax can favor education and hence growth at the expense of polluting activities. For example, in a model with a R&D sector reducing pollution, Grimaud and Tournemaine (2007) obtain this result as the tax increases the relative price of the polluting good. For finite lifetime, Pautrel (2011) finds that an increase in tax enhances growth as long as the abatement sector is more human capital intensive that the final output sector, while in Pautrel (2012) this result arises because pollution stems from physical capital.[3]

We depart from these papers in two major ways. First, we analyze an environmental policy with possible "instrument mixes". The government can implement a tax on pollution and recycle tax revenues in two types of environmental support: a direct one through public pollution abatement and a more indirect one through education subsidy.

Second, we assume that agents' preferences for the environment are endogenous. More precisely, we consider that green preferences are affected positively by both the individual human capital and the level of pollution, as supported by literature. A wide range of empirical studies identifies education as a relevant individual determinant of environmental preferences (see Blomquist and Whitehead, 1998; Witzke and Urfei, 2001 or European Commission, 2008). The intuition is that the more educated agent is, the more she is informed about environmental issues, and the more she

[2] See resolution 57/254 of United Nations General Assembly of 2002.

[3] Pautrel (2012) has a similar result than Gradus and Smulders (1993), in which pollution is due to physical capital, but it does not require that pollution affects directly education.

can be concerned about environmental protection. Likewise, environmental issues, as climate change or air pollution, harm welfare and push households to react. Among other explanations, pollution affects agents' well-being by damaging their health status (through mortality and morbidity) and by depreciating the environmental quality bequeathed to future generations. Schumacher (2009) highlights that when pollution is high, agents are more likely to be environmentally concerned and to act for the environment.

Considering an endogenous environmental awareness, our analysis is also related to the recent contribution of Prieur and Bréchet (2013), in which green preferences depend on human capital. The authors emphasize that the economy may be caught in a steady state without economic growth, while education policy can be used to achieve an asymptotic balanced growth path with sustained growth. Here, we extend this paper considering that education choices are not exogenous but stem from paternalistic altruism and that environmental preferences are driven by both human capital and pollution.[4]

In our overlapping generations model, growth is driven by human capital accumulation and environmental quality. Human capital depends on education spending chosen by altruistic parents, while the law of motion of the environment is in line with John and Pecchenino (1994). Production creates pollution flow, which damages environmental quality, whereas abatement activities improve it. To well identify consumer's environmental preferences, we use an impure altruism *à la* Andreoni (1990), where the contribution to the public good arises from private preferences for this good (pure altruism) and from a joy of giving. With this formalization, public and private contributions are no longer perfect substitutes, such that the controversial fiscal neutrality result of purely altruistic models does not hold.[5] Consequently, we consider two different incentives to explain pollution abatement: the level of environmental quality and the contribution itself to the environment.

With the present model, two regimes are distinguished: a regime with private contribution to pollution abatement and a regime without private contribution to this good. We provide conditions for the existence of a unique positive stable balanced growth path (hereafter BGP) with sustainable development. Depending on the share of public spending in public maintenance and the level of the tax, the BGP can be with or without private maintenance. We reveal that when the BGP is characterized by private pollution abatement, endogenous environmental concerns may generate damped oscillations, for low tolerance to pollution. Specifically, the feedback effect of human capital and environmental quality on green preferences influences the trade-off between private choices, which generates oscillations. Such complex dynamics leads to significant variations in the welfare across generations, which correspond to intergenerational inequalities along the convergence path (see Seegmuller and Verchère, 2004).

While the effect of environmental policy is generally studied in the long run, we underline that

[4]For a paper considering the effect of environmental quality on green concerns, see Schumacher and Zou (2013). Nevertheless, they assume that environmental quality has a discrete impact on preferences.

[5]In other words, when the government increases spending for the public good, there is not a complete crowding out effect on private contribution. Such impure altruism and non-neutrality are supported by empirical evidence. See Ribar and Wilhelm, (2002) or Crumpler and Grossman (2008) for a review on charitable giving and Menges et al (2005), on environmental contribution.

the short-term analysis represents also a crucial issue. Thus, two objectives of the government are identified: avoid oscillations in the short run, and achieve the highest growth rate in the long run. We emphasize that an increase in tax allows to address both short-and long-term objectives as long as the tax revenue is well allocated. More precisely, an intermediary allocation of the budget between public maintenance and education ensures that the economy converges to a BGP without private maintenance and with a sufficiently high support to education. Indeed, in this regime, environmental maintenance is entirely supported by public authorities. Consequently, there is no trade-off between private choices and oscillations never occurs. To achieve this regime, and hence avoid short-term issue, the share of budget in public maintenance has to be high enough. However, we show that an intermediary allocation of tax revenue is required to reach the highest long-term growth rate of both human capital and environmental quality. In this way, the budget devoted to pollution abatement is sufficiently high to be in the regime without private maintenance, where environmental quality is good, but education support is also sufficient so that the negative effect of tax on available income is more than offset by the positive effect of education subsidy.

The paper is organized as follows. In Section 2, we set up the theoretical model. Section 3 focus on the BGP and the transitional dynamics. In Section 4, we examine short-and long-term implications of environmental policy. Finally, Section 5 concludes. Technical details are relegated to an Appendix.

2 The model

Consider an overlapping generations economy, with discrete time indexed by $t = 0, 1, 2, ..., \infty$. Households live for two periods, childhood and adulthood, but take all decisions during their second period of life. At each date t, a new generation of N identical agents is born ($N > 1$). We assume no population growth.

2.1 Production

Production of the consumption good is carried out by a single representative firm. The output is produced according to a constant returns to scale technology:

$$Y_t = AH_t \tag{1}$$

where H_t is the aggregate stock of human capital and $A > 0$ measures the technology level. Defining $y_t \equiv \frac{Y_t}{N}$ as the output per worker and $h_t \equiv \frac{H_t}{N}$ as the human capital per worker, we have the following production function per capita: $y_t = Ah_t$.

The government collects revenues through a tax rate $0 \leqslant \tau < 1$ on production, which is the source of pollution. By assuming perfect competition, the profit-maximization problem yields the following factor price:

$$w_t = A(1 - \tau) \tag{2}$$

2.2 Consumer's behavior

Individual born in $t-1$ cares about her adult consumption level c_t, her child's human capital h_{t+1}, the current level of environmental quality Q_t and the future environment through altruism. To well identify these environmental preferences, we use an impure altruism in line with Andreoni (1990). Agent gets welfare for the total level of public good (i.e. the future environmental quality Q_{t+1}) but also from her action to contribute to this good (i.e. environmental maintenance m_t). With the joy of giving for m_t, public and private contributions are no longer perfect substitutes, such that the controversial neutrality result of purely altruistic models does not hold. Preferences are represented by the following utility function of a representative agent:

$$U(c_t, m_t, h_{t+1}, Q_{t+1}) = \ln c_t + \gamma_{1t} \ln(\varepsilon_1 \; m_t + \varepsilon_2 \; Q_{t+1}) + \gamma_2 \ln h_{t+1} + \gamma_3 \; {Q_t}^{\nu} \tag{3}$$

with γ_{1t}, γ_2, γ_3, ε_1, ε_2 and $\nu > 0$.

The parameter γ_3 captures taste for the current environmental quality. It corresponds to a usual well-being due to the environment but is taken as given by agent and not related with altruism concerns. The weight γ_2 is a paternalistic altruism factor for child's human capital, such that parents finance the child's education, as in Glomm and Ravikumar (1992).

The weight γ_{1t} captures the environmental awareness. We consider that these environmental preferences are affected negatively by the level of environmental quality (Q_t) and positively by the individual human capital (h_t), as supported by literature. Pollution, affecting welfare, has an impact on environmental behaviors: when pollution is high, agents are more likely to be concerned by the environment and to act in favor of it, as underlines Schumacher (2009). The worst environmental quality, the more the individual is able to realize the badness of the situation and therefore the more she has an incentive to protect the environment. At the same time, empirical behavioral economics literature identifies education as a determinant of the contribution to the environment (See Blomquist and Whitehead, 1998 or Witzke and Urfei, 2001). The economic intuition is that the more an agent is educated, the more she may be informed about environmental issues and their consequences, and thus the more she can be concerned about it. We assume that $\gamma_{1t} = \gamma_1(h_t, Q_t)$ where γ_1 is increasing and concave with respect to h, and decreasing and convex with respect to Q. In particular, we consider the following functional form:[6]

$$\gamma_{1t} \equiv \frac{\beta h_t + \eta Q_t}{\kappa h_t + Q_t} \tag{4}$$

with parameters β, κ, $\eta > 0$ and $\beta \geqslant \eta\kappa$. The parameter $-\kappa$ represents the lower bound of environment per unit of human capital, while β and η embody respectively the weight of human capital and of environmental in green awareness. Let us underline that when $\beta = \eta\kappa$, the environmental awareness is constant.

During childhood, individual does not make decisions. She is reared by her parents and benefits

[6]For a similar form, see Blackburn and Cipriani (2002) who use it to model the effect of pollution on longevity.

from education. When adult, she supplies inelastically one unit of labor remunerated at the wage w_t according to her human capital level h_t. She allocates this income to consumption c_t, education per child e_t and environmental maintenance m_t.[7] Furthermore, the government can subsidy education at the rate $0 \leqslant \theta_t^e < 1$, reducing the private cost of education. The budget constraint for an adult with human capital h_t is:

$$c_t + m_t + e_t(1-\theta_t^e) = w_t h_t \tag{5}$$

The human capital of the child h_{t+1} is produced with the private education expenditure e_t and the human capital of the parents h_t:

$$h_{t+1} = \epsilon\ {e_t}^{\mu} {h_t}^{1-\mu} \tag{6}$$

with $\epsilon > 0$, the efficiency of human capital accumulation. The parameter $0 < \mu < 1$ is compatible with endogenous growth and captures the elasticity of human capital to private education, while $1-\mu$ represents the share of human capital resulting from intergenerational transmission within the family.

The law of motion of environmental quality is defined by:

$$Q_{t+1} = (1-\alpha)Q_t + b(m_t + M_t + NG_t^m) - aY_t \tag{7}$$

where $\alpha > 0$ is the natural degradation of the environment and Y_t represents the pollution flow due to production in previous period. The parameter $a > 0$ corresponds to the emission rate of pollution, while $b > 0$ is the efficiency of environmental maintenance. The abatement activities are represented by a Cournot-Nash equilibrium approach. Each agent determines her own environmental maintenance (m_t), taking the others' contribution (M_t) as given. The government can provide public environmental maintenance $NG_t^m \geqslant 0$, which has the same efficiency than the private one. Following the seminal contribution of John and Pecchenino (1994), Q is an environmental quality index which can take positive or negative values and with an autonomous value of 0 in the absence of human intervention. This index may embody, for example, the inverse of the concentration of greenhouse gases in the atmosphere (like the chlorofluorocarbons, CFCs), or a local environmental public good such as the quality of groundwater in a specific area.

The consumer program is summarized by:

$$\max_{e_t, m_t}, U(c_t, m_t, h_{t+1}, Q_{t+1}, Q_t) = \ln c_t + \gamma_{1t} \ln(\varepsilon_1\ m_t + \varepsilon_2\ Q_{t+1}) + \gamma_2 \ln h_{t+1} + \gamma_3\ Q_t^{\nu} \tag{8}$$

$$s.t \quad c_t + m_t + e_t(1-\theta_t^e) = w_t h_t$$

$$h_{t+1} = \epsilon\ {e_t}^{\mu} {h_t}^{1-\mu}$$

$$Q_{t+1} = (1-\alpha)Q_t + b(m_t + M_t + NG_t^m) - aY_t$$

with $m_t \geqslant 0$.

[7] See Kotchen and Moore (2008) for empirical evidences of private provision of environmental public goods.

2.3 The government

The design of environmental policy represents a major challenge for governments. Among other reports, OECD (2007 and 2008) recommends the revenue recycling of tax on polluting activities in order to complete the governmental action. This kind of policy is observable in several countries. For example, in France, the government implements a general tax on polluting activities (TGAP) and transfers revenues to the French Environment and Energy Management Agency (ADEME) that funds activities in favor of environment. While environmental policy is often studied through taxation in theoretical literature, there exist several policy levers. Policy makers can support direct environmental actions (e.g. conservation of forests and soils, water treatment, waste management), but also more indirect actions attempting to change behaviors (e.g. environmental education).

In this model, in order to study such environmental policy, we consider the following policy scheme. Since pollution is a by-product of the production process, the government taxes the output at rate τ and the public budget is spent on public environmental maintenance NG_t^m or/and on education subsidy θ_t^e. The government's budget is balanced at each period, such that:

$$N(\theta_t^e e_t + G_t^m) = \tau Y_t \tag{9}$$

We define the share of public expenditure devoted to public maintenance $0 \leqslant \sigma \leqslant 1$, and to education subsidy $(1-\sigma)$, assumed constant:

$$\sigma = \frac{NG_t^m}{\tau Y_t} \;\; ; \;\; 1-\sigma = \frac{N\theta_t^e e_t}{\tau Y_t} \tag{10}$$

Thus, fiscal policy is summarized by two instruments $\{\tau;\, \sigma, takenasgivenbyconsumers.$

2.4 Equilibrium

The maximization of the consumer program (8) leads to the following first order conditions on education expenditure and on environmental maintenance:

$$\frac{\partial U}{\partial e_t} = 0 \Leftrightarrow \frac{1-\theta_t^e}{c_t} = \frac{\gamma_2 \mu}{e_t} \tag{11}$$

$$\frac{\partial U}{\partial m_t} \leqslant 0 \Leftrightarrow \frac{1}{c_t} \geqslant \frac{\gamma_{1t}(\varepsilon_1 + \varepsilon_2 b)}{\varepsilon_1 m_t + \varepsilon_2 Q_{t+1}} \tag{12}$$

From equations (5), (7) and the first order conditions (11) and (12), we deduce the optimal choices in terms of education and maintenance in two regimes: an interior solution, where individuals invest in environmental protection $m_t > 0$ (hereafter pm) and a corner solution without private contribution to the environment $m_t = 0$ (hereafter npm).

$$e_t = \begin{cases} \left(\frac{\gamma_2 \mu}{1-\theta_t^e}\right)\left(\frac{(\varepsilon_1+\varepsilon_2 b)w_t h_t + \varepsilon_2\left[(1-\alpha)Q_t - aY_t + b(M_t + NG_t^m)\right]}{(1+\gamma_{1t}+\gamma_2\mu)(\varepsilon_1+\varepsilon_2 b)}\right) & pm \\ \frac{w_t h_t \gamma_2 \mu}{(1+\gamma_2\mu)(1-\theta_t^e)} & npm \end{cases} \tag{13}$$

$$m_t = \begin{cases} \frac{\gamma_{1t}(\varepsilon_1+\varepsilon_2 b)w_t h_t - \varepsilon_2(1+\gamma_2\mu)\left[(1-\alpha)Q_t - aANh_t + b(M_t+NG_t^m)\right]}{(1+\gamma_{1t}+\gamma_2\mu)(\varepsilon_1+\varepsilon_2 b)} & pm \\ 0 & npm \end{cases} \tag{14}$$

At the symmetric equilibrium, $M_t = m_t(N-1)$, the wage equilibrium is $w_t = A(1-\tau)$, the production function is $Y_t = ANh_t$ and the government budget constraint is given by (9). The Nash intertemporal equilibria are thus given by:

$$e_t = \begin{cases} \frac{\gamma_2\mu[(\varepsilon_1+\varepsilon_2 bN)A(1-\tau)h_t+\varepsilon_2((1-\alpha)Q_t+ANh_t(b\sigma\tau-a))]}{\gamma_{1t}(\varepsilon_1+\varepsilon_2 b)+(1+\gamma_2\mu)(\varepsilon_1+\varepsilon_2 bN)} + (1-\sigma)\tau Ah_t & pm \\ \frac{Ah_t[\gamma_2\mu(1-\tau)+\tau(1-\sigma)(1+\gamma_2\mu)]}{1+\gamma_2\mu} & npm \end{cases} \tag{15}$$

$$m_t = \begin{cases} \frac{\gamma_{1t}(\varepsilon_1+\varepsilon_2 b)Ah_t(1-\tau)-\varepsilon_2(1+\gamma_2\mu)[(1-\alpha)Q_t+ANh_t(b\sigma\tau-a)]}{\gamma_{1t}(\varepsilon_1+\varepsilon_2 b)+(1+\gamma_2\mu)(\varepsilon_1+\varepsilon_2 bN)} & pm \\ 0 & npm \end{cases} \tag{16}$$

Education spending depends positively on environmental quality. The better the environment, the lower the optimal amount of maintenance activities, as a result, individual can devote more resources to educate her child.

The public policy instruments shape education and abatement spendings differently. An increase in tax implies a negative income effect (wage decreases) but still favors education spending when public expenditure is sufficiently devoted to education subsidies (σ low).[8] Conversely, an increase in tax always affects negatively maintenance activities. In addition to the negative income effect, the tax increases the public pollution abatement which crowds out private maintenance. Nevertheless, public spending substitutes only partially to the private one due to the direct benefit from contribution to the environment.

Let us introduce, for the rest of the paper, a green development index X_t, equal to environmental quality per unit of human capital: $X_t \equiv \frac{Q_t}{h_t}$.

Note that, due to the log-linear utility, $\varepsilon_1 m_t + \varepsilon_2 Q_{t+1}$ has to be positive. This condition can be expressed as:

$$X_t > \begin{cases} \frac{-A[\varepsilon_2 N(b(1-\tau)+b\tau\sigma-a)+\varepsilon_1(1-\tau)]}{\varepsilon_2(1-\alpha)} \equiv X^{\exists} & pm \\ 0 & npm \end{cases} \tag{17}$$

The environmental awareness, given by (4), can be rewritten in terms of X:

$$\gamma_{1t} = \frac{\beta + \eta X_t}{\kappa + X_t} \tag{18}$$

The parameter $-\kappa$ represents a critical threshold of pollution as a limit not to exceed, with respect to the individual human capital. The higher κ is, the higher is the tolerated level of pollution per unit of human capital (i.e. the lower X can be). Thus we will also refer to κ as the tolerance to pollution. To examine the implications of the critical threshold, we set $-\kappa \geqslant X^{\exists}$, and to ensure that $\gamma_{1t} > 0$, we consider $X_t > -\kappa$.

[8] $\frac{\partial e_t}{\partial \tau} > 0$ $\left(resp.\ \frac{\partial e_t}{\partial \tau} < 0\right)$, when $\sigma < \frac{\gamma_{1t}c_1+c_3}{\gamma_{1t}c_1+c_3+\varepsilon_1\gamma_2\mu}$ $\left(resp.\ 1 \geqslant \sigma > \frac{\gamma_{1t}c_1+c_3}{\gamma_{1t}c_1+c_3+\varepsilon_1\gamma_2\mu}\right)$.

Using equation (16), we deduce the following central property. The solution without private environmental maintenance occurs when:

$$X_t \geqslant \frac{A\left[\gamma_{1t}(\varepsilon_1+\varepsilon_2 b)(1-\tau)-\varepsilon_2 N(1+\gamma_2\mu)(b\sigma\tau-a)\right]}{\varepsilon_2(1+\gamma_2\mu)(1-\alpha)} \tag{19}$$

In this case, the level of environmental quality is so high and/or the level of human capital so low, that the private abatement of pollution is given up. Policy favors the occurrence of this regime. When τ is positive, the economy is more likely to be characterized by no maintenance activities, especially if σ is positive, since public maintenance partially substitutes for private one. But when revenue recycling is entirely devoted to education ($\sigma = 0$), agent still replaces some abatement activities by education, as the last becomes relatively less costly. The condition (19) can be written as:

$$\begin{aligned}\mathcal{P}(X_t) \quad \equiv \quad & X_t^2\varepsilon_2(1+\gamma_2\mu)(1-\alpha)+X_t[\varepsilon_2(1+\gamma_2\mu)(\kappa(1-\alpha)+AN(b\sigma\tau-a))-A\eta c_1(1-\tau)] \\ & -A[\beta c_1(1-\tau)-\kappa(b\sigma\tau-a)N\varepsilon_2(1+\gamma_2\mu)] \geqslant 0\end{aligned} \tag{20}$$

$\mathcal{P}(X_t) = 0$ admits one solution, $X_t = \Lambda \geqslant 0$.[9] Thus, households invest in environmental protection when $X_t \in (-\kappa, \Lambda)$.

Using the human capital accumulation (6) and the environmental quality process (7), and the first order conditions (15) and (16), we finally obtain the dynamic equation characterizing equilibrium paths:

Definition 1. *Given the initial condition* $X_0 = \frac{h_0}{Q_0} > -\kappa$*, the intertemporal equilibrium is the sequence* $(X_t)_{t\in\mathbb{N}}$ *which satisfies, at each* t, $X_{t+1} = \mathcal{F}(X_t)$*, with:*

$$\mathcal{F}(X_t) = \begin{cases} \frac{(1-\alpha)\ X_t\ [\gamma_{1t}\ c_1+c_2]+AN\ [\gamma_{1t}\ c_1 b(1-\tau)+(\gamma_1 c_1+c_2)(b\tau\sigma-a)]}{\epsilon[\gamma_2\mu A\ c_3(1-\tau)+\gamma_2\mu\varepsilon_2[(1-\alpha)X_t+AN(b\sigma\tau-a)]+(1-\sigma)\tau A(\gamma_{1t}\ c_1+(1+\gamma_2\mu)c_3)]^{\mu}[\gamma_{1t}\ c_1+(1+\gamma_2\mu)c_3]^{1-\mu}} & pm \\ \frac{(1-\alpha)X_t+AN(b\sigma\tau-a)}{\epsilon\left[\frac{\gamma_2\mu A(1-\tau)+(1-\sigma)\tau A(1+\gamma_2\mu)}{1+\gamma_2\mu}\right]^{\mu}} & npm \end{cases} \tag{21}$$

We define the endogenous growth rate of human capital g_H, and environmental quality g_Q, using equations (6), (7), (15) and (16):

$$1+g_{Ht} = \begin{cases} \epsilon\left[\frac{\gamma_2\mu[A(1-\tau)c_3+\varepsilon_2((1-\alpha)X_t+AN(b\sigma\tau-a))]+(1-\sigma)\tau A(\gamma_{1t}c_1+c_3(1+\gamma_2\mu))}{\gamma_{1t}c_1+(1+\gamma_2\mu)c_3}\right]^{\mu} & pm \\ \epsilon\left[\frac{A[\gamma_2\mu(1-\tau)+\tau(1-\sigma)(1+\gamma_2\mu)]}{1+\gamma_2\mu}\right]^{\mu} & npm \end{cases} \tag{22}$$

where c_1, c_2 and c_3, three positive constants defined by $c_1 \equiv \varepsilon_1 + \varepsilon_2 b$; $c_2 \equiv \varepsilon_1(1+\gamma_2\mu)$ and $c_3 \equiv \varepsilon_1 + \varepsilon_2 bN$. In the *npm* regime, human capital growth rate is constant, as education spending does not depend on the environment. The *pm* regime is characterized by a human capital growth rate increasing in the green development index, directly and indirectly though environmental awareness γ_{1t}.

[9] We have $\Lambda > 0$ when $\beta c_1(1-\tau) - \kappa(b\sigma\tau - a)N\varepsilon_2(1+\gamma_2\mu) \geqslant 0$ and $\Lambda = 0$ otherwise.

$$1+g_{Qt}=\begin{cases}\frac{(1-\alpha)X_t(\gamma_{1t}c_1+c_2)+AN[\gamma_{1t}\ c_1(1-\tau)b+(\gamma_{1t}c_1+c_2)(b\tau\sigma-a)]}{X_t(\gamma_{1t}c_1+c_3(1+\gamma_2\mu))} & pm\\ 1-\alpha+\frac{AN(b\sigma\tau-a)}{X_t} & npm\end{cases} \tag{23}$$

In the case with private maintenance, the green development index X_t has a direct negative impact on the growth rate of environmental quality and an indirect positive effect through environmental awareness γ_{1t}. In the corner solution, g_{Qt} is always negative without public abatement.[10] However, when the government intervenes, the growth of the environmental quality at the corner may be positive for sufficiently high share of policy devoted to public environmental maintenance (σ).

From (22) and (23), we make the following assumption in order to guarantee that positive growth rates of environmental quality and human capital are possible when X is positive.[11]

Assumption 1. *Let* $\sigma_{Min}(\tau)\equiv\frac{a(\eta c_1+c_2)-b\eta c_1(1-\tau)}{b\tau(\eta c_1+c_2)}$ *and* $\sigma_{Max}(\tau)\equiv\frac{A\gamma_2\mu+A\tau-\epsilon^{-1/\mu}(1+\gamma_2\mu)}{A\tau(1+\gamma_2\mu)}$. *We assume:*

a) $\gamma_2\mu A\epsilon^{\frac{1}{\mu}}>1+\gamma_2\mu$ *and* $\frac{b}{a}>\max\left\{\frac{\eta c_1+c_2}{\eta c_1},\frac{1}{1+\gamma_2\mu}\right\}$

b) $\sigma_{Min}(\tau)<\sigma<\sigma_{Max}(\tau)$.[12]

This Assumption has to be satisfy otherwise environmental quality index and/or human capital go down, and the economy always collapses. The Assumption 1.a implies that human capital accumulation and environmental maintenance are sufficiently efficient.[13] Note that the second inequality of Assumption 1.a entails that $b>a$, which is relevant since maintenance is devoted to the protection of the environment, contrary to pollution flow which is only a residual of the production process, as Mariani et al. (2010) argue. The Assumption 1.b restrains some policy schemes. For low level of tax, all allocation σ are possible, while for high level, extreme allocations between public spendings are excluded.

3 Balanced growth path and transitional dynamics

We examine in this section the existence of a BGP equilibrium characterized as:

Definition 2. *A balanced growth path (BGP) satisfies Definition 1 and has the following additional properties: the stock of human capital and environmental quality grow at the same and constant rate* g_i*, with subscripts* $i=\{pm,npm\}$ *denoting respectively the regime with Private Maintenance and the regime where there is No Private Maintenance. This equilibrium path is such that the green development index* X_t *is constant and defined by* $X_{t+1}=X_t=\bar{X}_i$.

[10] When $m_t=0$ and $\sigma=0$, g_{Qt} is increasing in X_t: an increase in X_t fits in with a human capital decline and hence with a lower pollution.

[11] The assumption is set so that it is valid for all values that γ_{1t} can take across time.

[12] Under Assumption 1.a, $\sigma_{Min}(\tau)$ is increasing in τ from $\lim_{\tau\to0}\sigma_{Min}=-\infty$ to $\lim_{\tau\to1}\sigma_{Min}=a/b$, while $\sigma_{Max}(\tau)$ is decreasing in τ from $\lim_{\tau\to0}\sigma_{Max}=+\infty$ to $\lim_{\tau\to0}\sigma_{Max}=1-\frac{1}{\epsilon^{1/\mu}A}$.

[13] This condition also implies that the threshold level ensuring the existence $X^{\exists}$, expressed in (17), is negative.

From Definitions 1 and 2 and equations (18) and (20), we emphasize the properties of the dynamic equation characterizing equilibrium paths, $\mathcal{F}$:

Lemma 1. *Under Assumption 1 and for $\beta > \eta\kappa$:*

- *When $X < \Lambda$, $\lim_{X\to-\kappa}\mathcal{F}(X) = \frac{AN(b(1-\tau)+b\tau\sigma-a)-(1-\alpha)\kappa}{\epsilon[A(1-\sigma)\tau]^{\mu}}$.*

  i) If $N > \bar{N}$, we have $\lim_{X\to-\kappa}\mathcal{F}(X) > -\kappa$.

  ii) If $N \leqslant \bar{N}$, we have $\lim_{X\to-\kappa}\mathcal{F}(X) \leqslant -\kappa$.

  $\mathcal{F}(0) > 0$ and $\lim_{X\to\Lambda^-}\mathcal{F}(X) = v$.

- *When $X \geqslant \Lambda$, $\mathcal{F}$ is increasing and linear in X, $\mathcal{F}(\Lambda) = v$ and $\lim_{X\to+\infty}\mathcal{F}(X) = +\infty$.*

with $\bar{N} = \frac{\kappa(1-\alpha-\epsilon(A(1-\sigma)\tau)^{\mu})}{A[b(1-\tau)+b\tau\sigma-a]}$ and v is a positive constant.

Proof. See Appendix 6.1. ∎

We focus on the case where population is not too low:

Assumption 2 $N > \bar{N}$

The extreme case $N \leqslant \bar{N}$ is of minor interest in our model because the overall reaction of agents would never be sufficient to improve the environment when the environmental quality tends to its critical level ($-\kappa$), whatever individual's efforts.[14]

From Lemma 1, we deduce the existence of the BGP $\bar{X}_i$ corresponding to the solutions of equation $\mathcal{F}(\bar{X}_i) = \bar{X}_i$, where $\mathcal{F}(\bar{X}_i)$ is obtained with equations (18) and (21):

Proposition 1. *Under Assumptions 1, 2 and $\beta > \eta\kappa$, there exists a unique positive BGP ($\bar{X}_i$), such that, according to a critical threshold $\hat{\sigma}(\tau)$:*

- *When $\sigma > \hat{\sigma}(\tau)$, the BGP is in the regime without private maintenance (npm).*

- *When $\sigma < \hat{\sigma}(\tau)$, the BGP is in the regime with private maintenance (pm).*

where $\hat{\sigma}(\tau)$ is a decreasing function of τ, with $\lim_{\tau\to 0}\hat{\sigma}(\tau) = +\infty$ and $\lim_{\tau\to 1}\hat{\sigma}(\tau) = a/b$.

Proof. See Appendix 6.2. ∎

The policy makes possible a sustainable BGP equilibrium without private abatement, according to the share of public spending devoted to environmental protection (σ). If it is sufficiently high, households may stop investing in private maintenance in long run, as underlined in Proposition 1. However, when environmental awareness (γ_1) is too high, the tax rate (τ) required to the existence of the regime without private maintenance is high.

[14] The reverse assumption does not change our results. It would only make possible the existence of traps for extreme values of X_0.

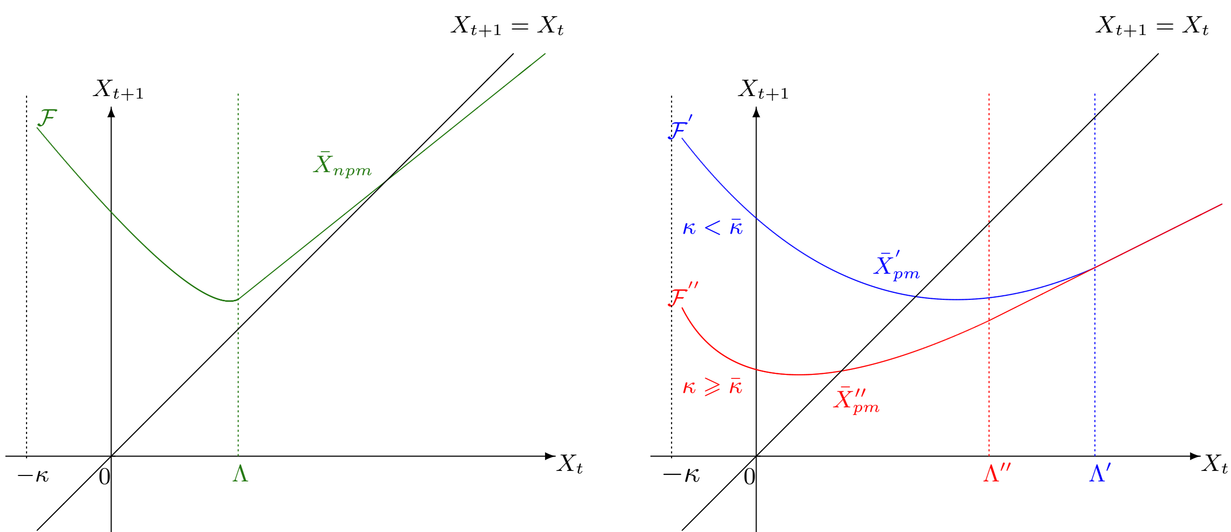
We observe that endogenous green preferences may lead to extreme behaviors. When environmental quality tends to its critical threshold $-\kappa$, green preferences soar implying a huge effort in maintenance at the expense of education. The significant improvement of X allows to catch up the lag in human capital and to achieve a sustainable development. However, as long as a part of public revenue is devoted to education ($\sigma < 1$), policy smooths extreme behaviors in terms of maintenance, as the government compels agents to invest sufficiently in education.

From Lemma 1, we derive the stability properties of the BGP presented in Proposition 1. When the BGP is in the regime without private maintenance, we obtain an explicit solution whose dynamics is easily deduced. To analyze the stability of the equilibrium in the regime with private maintenance, we normalized $\bar{X}_{pm}$ to one, using the scaling parameter ϵ. The results are then appraised in the following proposition and illustrated in Figure 1 :

Proposition 2. *Under Assumptions 1, 2, and $\beta > \eta\kappa$:*

- *The BGP in the regime without private maintenance, $\bar{X}_{npm}$, is globally and monotonously stable.*
- *The BGP in the regime with private maintenance, $\bar{X}_{pm}$, is locally stable and there exists a $\bar{\kappa}$ such that:*
  - *for $\kappa \geqslant \bar{\kappa}$: the convergence is monotonous.*
  - *for $\kappa < \bar{\kappa}$: Without education policy, there exists a $\tilde{\beta}$ such that the dynamics is oscillatory (resp. monotonous) when $\beta > \tilde{\beta}$ (resp. $\beta < \tilde{\beta}$). With education policy, the convergence may be oscillatory for intermediary values of β.*

Proof. See Appendix 6.3. ■



(a) *when $\sigma > \hat{\sigma}(\tau)$* **(b)** *when $\sigma < \hat{\sigma}(\tau)$*

Figure 1: *Dynamics*

As underlined in Proposition 2, the economy may display damped oscillations because of endogenous concerns.[15] The emergence of complex dynamics is explained by the feedback effect between green development index and environmental awareness, which affects the trade-off between education and maintenance. In the absence of private maintenance, this trade-off does not exist and the dynamics is always monotonous. Whereas when agent invests in environmental protection, cyclical convergence may occur and can be described as follows: an increase in γ_{1t} encourages private maintenance investment at the expense of education spending. For the next generation, it generates a fall in h_{t+1}, a raise in Q_{t+1}, and so a decrease in γ_{1t+1}. These modifications entail multiple effects on the private choices: they all shape negatively m_{t+1} whereas for e_{t+1} the impact is ambiguous. Indeed, education spending is affected positively by the improvement of environmental quality Q_{t+1} and by the decrease in the green preferences γ_{1t+1}, while the fall in human capital h_{t+1} entails a negative income effect. As long as the positive impacts on education exceed the negative income effect, the economy displays oscillations. The opposite variation in h_{t+1} acts as a brake on oscillations, i.e. a stabilizing effect, such that cyclical variations are damped.

Oscillatory dynamics is observed only when agent's tolerance to pollution (κ) is sufficiently low. More precisely, in the absence of public intervention in education ($\sigma = 1$), the economy displays oscillations for low tolerance to pollution (κ) and high sensitivity of concerns to human capital (β). In this case, green preferences are more sensitive to variation in X.[16] Thus, environmental awareness (γ_1) experiences important variations along the converging trajectory, and so do levels of environmental quality and human capital.

As long as the government subsidies education ($\sigma \in [0, 1)$), the conditions to observe oscillatory cases are less obvious. Environmental awareness does not affect the trade-off between education and private maintenance in the same way, as a part of private education spending is now independent on agent's preferences. For low β, as previously, agent's green preferences are not sensitive enough to variation in X to generate complex dynamics. However, when green preferences are highly sensitive to human capital, the result is more complex. A very high β implies an important environmental awareness, such that agents greatly value maintenance at the expense of human capital and education is mainly driven by policy. Thus, variations in private choices are smoothed and the economy may display damped oscillations only for intermediate level of β.[17]

Our results in transitional dynamics are opposed to Zhang (1999), who finds that greener preferences are necessary to avoid complicated dynamic structure.[18] Instead, they are close to Ono (2003b), who argues that concerns for the environment would cause oscillations. However, his mechanism goes through innovation and corresponds to higher levels of exogenous green preferences, while in our setup such dynamics arises from the endogenization of environmental awareness and the feedback effect of

[15] At the limit case $\beta = \eta\kappa$, γ_1 is exogenous and the dynamics is always monotonous. The proof is available upon request.

[16] Function $\gamma_1(X_t)$ is more convex in X_t when κ is low and/or β is high.

[17] When public budget is mainly devoted to maintenance (σ high), damped oscillations always occur for intermediate β.

[18] He develops a model *à la* John and Pecchenino (1994) to study the cases of nonlinear dynamics and endogenous fluctuations.

environment and education on green behaviors.

When economy experiences damped oscillations, the convergence toward the BGP is characterized by oscillations in the levels and the growth rates of human capital and environmental quality. Generations with high environmental quality over human capital ratio are followed by generations with low environmental quality over human capital. As Seegmuller and Verchère (2004) show, such cyclical convergence make the welfare varies across generations and corresponds to intergenerational inequalities.[19]

4 Environmental policy implications

In this section, we analyze the implications of environmental policy. More precisely, we attempt to emphasize what policy can allow to avoid intergenerational inequalities in the short-run and to enhance the long-term growth rate.

4.1 The short-term effect of environmental tax

We point out, previously, that the economy may exhibit complex dynamics when environmental awareness is endogenous. We wonder then how a tighter environmental tax affects this short-term situation and how the government should use it to reduce intergenerational inequalities. Focusing on the BGP in the regime with private maintenance, where damped oscillations may occur, we examine the effect of an increase in environmental tax on transitional dynamics:

Proposition 3. *Under Assumptions 1, 2 and $\beta > \eta\kappa$, when there is private maintenance at the stable BGP, an increase in τ implies that:*

- *If the BGP remains in the regime with private maintenance ($\sigma < \hat{\sigma}(\tau)$), there exists a $\tilde{\sigma} \in (0,1)$ such that:*
  - *For $\sigma < \tilde{\sigma}$, the range of κ associated with damped oscillations decreases.*
  - *For $\sigma > \tilde{\sigma}$, the range of κ associated with damped oscillations increases.*
- *If the BGP moves to the regime without private maintenance ($\sigma > \hat{\sigma}(\tau)$), there is no damped oscillations.*

Proof. See Appendix 6.4. ■

From Proposition 3, we highlight that the government intervention can neutralize or generate damped oscillations. It comes from the fact that policy shapes the trade-off between maintenance and education spendings, and hence the mechanism driving oscillations.

When $\sigma < \min\{\tilde{\sigma}; \hat{\sigma}(\tau)\}$, an increase in environmental tax allows to reduce the case where oscillations arise. As highlighted in Section 3, cyclical convergence may occur since green preferences

[19] Such complex dynamics can also illustrate economic volatility, as Varvarigos (2011) emphasizes.

are endogenous. In this situation, along the convergence path, education variations stem from several effects through Q, γ_1 and h. The former effects drive oscillations while the latter works in the reverse. As long as σ is low enough, a tighter tax reinforces the impact of human capital on private education spending. Indeed, the fall in wage, entailed by tax, is overcompensated by the increase in education subsidy. Education spending is mainly driven by the government's action and hence become less sensitive to γ_1. As a result, oscillatory trajectories are less frequent, i.e. the critical level of tolerance to pollution ($\bar{\kappa}$) under which damped oscillations may occur is lower.

When $\tilde{\sigma} < \sigma < \hat{\sigma}(\tau)$, public revenue devoted to public abatement is high but not enough to be at the regime without private maintenance. Unlike to the previous case, σ is high enough such that education subsidy is too low to compensate the decrease in wage. Thus, environmental tax diminishes the influence of human capital on private education spending and oscillations occur more frequently. Specifically, the critical level of tolerance to pollution ($\bar{\kappa}$) under which damped oscillations may be observed is higher.

In the previous section, we emphasize that an increase in environmental tax makes the regime without private maintenance (*npm*) more frequent ($\hat{\sigma}(\tau)$ goes down) and that the convergence to the BGP in this regime is always monotonous. Thus, the government may also avoid intergenerational inequalities by fixing a sufficiently high tax on pollution and devoting a large share of public spending to maintenance. For a BGP initially in the regime with private maintenance (*pm*), if $\hat{\sigma}(\tau)$ becomes lower than σ, the BGP moves to the regime without private maintenance, there is no more trade-off between private choices and hence transitional dynamics does not result in oscillations.

Thus, from Proposition 3, we point out that with endogenous concerns, the environmental tax may favor intergenerational inequalities around the BGP when the product of the tax is not correctly used. The implications of environmental policy in the short-run are rarely studied. A notable exception is Ono (2003b), who emphasizes that a sufficient increase in environmental tax may shift the economy from a fluctuating regime to a BGP where capital and environmental quality go up perpetually. In this paper, we highlight that the use of tax revenue is decisive for such a change.

4.2 The long-term effect of environmental tax

In accordance with the concept of sustainable development[20], the environment is also and above all a long-term concern. In this respect, we want to study what solutions the government can put in place to achieve higher long-term growth of both human capital and environmental quality. Thus, we examine how policy affects the stable BGP and the corresponding growth rate.

A tighter environmental policy influences the long-term growth rate through several channels. First directly, by affecting the trade-off between education and maintenance activities. Second indirectly, by modifying the green development index and environmental preferences.[21] Therefore, the

[20] The Brundtland Commission (WCED, 1987) defines the sustainable development as "development that meets the needs of the present without compromising the ability of future generations to meet their own needs".

[21] Note that examining the impact of environmental awareness component on growth, we obtain that the stronger environmental concerns, the lower the growth rate, as in Prieur and Bréchet (2013). In their paper, a raise in environmental awareness always reduces physical capital accumulation. Here, γ_1 affects growth trough an additional channel as

global impact is ambiguous. In the following lemma, we investigate how authorities can improve the growth rate along the stable BGP:

Lemma 2. *Under Assumptions 1, 2 and $\beta > \eta\kappa$, following an increase in τ:*

- *When the BGP is initially in the npm regime ($\sigma > \hat{\sigma}(\tau)$), the growth rate is enhanced for $\sigma < \frac{1}{1+\gamma_2\mu}$.*
- *When the BGP is initially in the pm regime ($\sigma < \hat{\sigma}(\tau)$):*
  - *If it remains in the pm regime ($\sigma < \hat{\sigma}(\tau)$), there exists an interval $(\underline{\sigma}(\tau), \overline{\sigma}(\tau))$ such that the growth rate goes up for $\underline{\sigma}(\tau) < \sigma < \overline{\sigma}(\tau)$.*
  - *If it moves to the npm regime ($\sigma > \hat{\sigma}(\tau)$), the growth rate is enhanced and is higher than in the pm regime when $\sigma < \frac{1}{1+\gamma_2\mu}$.*

Proof. See Appendix 6.5 ■

From Lemma 2, we deduce the following proposition:

Proposition 4. *Under Assumptions 1, 2 and $\beta > \eta\kappa$, for a given $\sigma \in \left(\frac{a}{b}; \frac{1}{1+\gamma_2\mu}\right)$:*

- *The growth rate is higher in the npm regime than in the pm one.*
- *The growth rate in the npm regime is enhanced with a tighter tax.*

Proof. See Appendix 6.5 ■

Considering an increase in τ, an economy initially with private maintenance may switch to the other regime. When the BGP remains in the regime with private maintenance ($\sigma < \hat{\sigma}(\tau)$), an increase in tax favors both human capital and environmental quality if σ is intermediary. When σ is too low, policy favors mainly education spending. Despite the improvement in γ_1 it entails, the increase in tax makes the private maintenance too expensive, such that the environment may deteriorate, and so does the growth rate. Conversely, when σ is too high, the tax revenue contributes mostly to public maintenance. Even if the private investment in environment diminishes (in favor of education spending), education cost is too high, which may weaken human capital accumulation. Thus, authorities can increase growth by fixing intermediary value of σ.

When the BGP is or moves to the regime without private maintenance ($\sigma > \hat{\sigma}(\tau)$),[22] a tighter tax leads to the highest growth rate as long as it is accompanied by a sufficient support for human capital ($\sigma < 1/(1 + \gamma_2\mu)$). The intuition is the following. On one hand, in the *npm* regime, maintenance is entirely public despite the willingness of agent to contribute privatively to pollution abatement. Therefore, the environmental quality is sufficiently good at this regime. On the other hand, when

the environment improves education. Nevertheless, the negative direct impact of environmental awareness on education more than offsets the improvement of the environment. The proof is available upon request.

[22]Following an increase in τ, a BGP initally in the *npm* regime cannot move to the *pm* regime

the government allocates a high enough share of its budget to education, the negative effect of tax on available income is more than offset by the positive effect through education subsidy. In this way, human capital accumulation and the environment are enhanced.

Note that when $\sigma > 1/(1+\gamma_2\mu)$, a regime switch can be growth-reducing, particularly if the increase in τ moving the BGP to the regime without private maintenance is important. In this case, the negative income effect exceeds the positive impact of education subsidy and hence human capital accumulation deteriorates.

Our results contribute to the literature on environmental policy and growth. As in Ono (2003a, 2003b), environmental taxation exerts competing effects on the long-run economic growth. However, while he observes a positive relationship between the tax and the long-term growth only for intermediary level of the tax rate, we emphasize that a tighter policy is always growth-enhancing as long as the tax revenue is well allocated. The relationship that we observe between tax and long-term growth rate is also tied to results obtained in a recent branch of this literature, that considers the role of human capital. In a model with a R&D sector reducing pollution, Grimaud and Tournemaine (2007) find that a higher environmental tax decreases the price of education relatively to the polluting good, such that a tighter tax always promotes growth. The same link is present in Pautrel (2011, 2012) when lifetime is finite. In the first paper, this holds when abatement sector is more human capital intensive than final output sector, while in the second this is due to the fact that pollution stems from physical capital. In these three studies, the underlying mechanism is that an increase in tax favors education at expense of polluting activities. We differ from them by pointing out the role of policy mix. The government can implement a win-win policy only if it allocates properly the revenue of the environmental tax between education subsidy and public pollution abatement.

We emphasize that the recycling of environmental tax can have double dividend: environmental quality improvement and economic benefit. However, we differ from the literature on the "double dividend hypothesis" which considers the economic dividend as an improvement in economic efficiency from the use of environmental tax revenue to reduce other distortionary taxes (e.g. Goulder, 1995 or Chiroleu-Assouline and Fodha, 2006). Instead, we emphasize that, when the tax on polluting activities is recycled in public maintenance and education subsidy, policy can promote both environmental quality and human capital.

4.3 How the government can meet the short-and long-term objectives?

We emphasize previously that, in the short run the government should intervene to avoid intergenerational inequalities, while in the long run it seeks to enhance growth. In this section, we examine how a tighter tax can satisfy these two objectives. By considering the properties of the function $\hat{\sigma}(\tau)$, that defines the regimes with and without private maintenance, with the properties of $\sigma_{Min}(\tau)$ and $\sigma_{Max}(\tau)$, that determine the possible policy schemes (see Assumption 1), we summarize the results of Propositions 3 and 4:

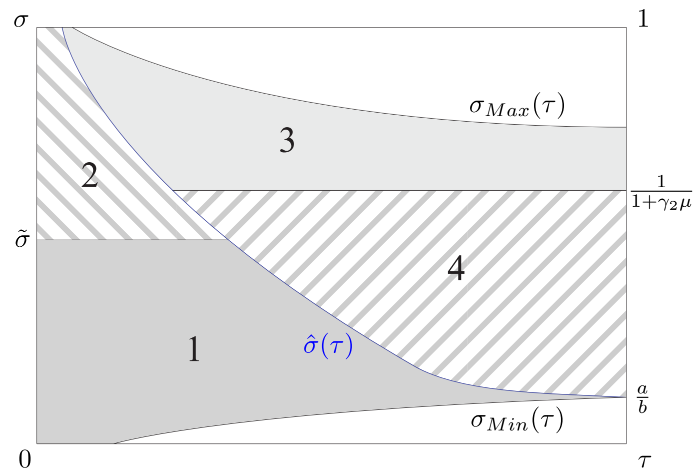


Figure 2: *Short-and long-term implications of a tighter tax, at σ given*

This figure depicts the implications of a tighter tax for a given σ. Fourth areas are distinguished.

In areas 1 and 2, the BGP equilibrium is in the regime with private maintenance. Focusing on the short run, a tighter tax makes the occurrence of oscillations less likely when $\sigma < \tilde{\sigma}$ (area 1), while the opposite holds when $\sigma > \tilde{\sigma}$ (area 2). In these areas, to achieve the long-term objective, an increase in the pollution tax has to be associated with an intermediary level of σ. Unfortunately, no clear conclusion emerges when comparing these values of σ with the one corresponding to benefit in the short-run.

As long as the tax rate is sufficiently high, the economy can achieve areas 3 and 4, where the BGP is in the regime without private maintenance and hence contributions to the pollution abatement is entirely public. At this state, the short-term issue vanishes and the economy performs a higher long-term growth rate when education subsidy is sufficiently high (σ low, area 4). Furthermore, as stressed in Proposition 4, for a given σ, the growth rate is higher in area 4 than in the other regime.

As a result, we identify the most favorable tax scheme as the one where the environmental protection results exclusively in public spending and education support is sufficiently high. In such a way, there is no more issue about the trade-off between education and maintenance, which would lead eventually to damped oscillations and hence to inequalities. Moreover, since public maintenance and education subsidy are sufficiently high, human capital accumulation is favored without damaging the environment.

Note that the design of this policy depends on environmental awareness. For a given σ, the higher green preference parameters, the higher the tax rate required to achieve the area 4 (the curve $\hat{\sigma}(\tau)$ moves to the right).

5 Conclusion

In this paper, we examine the implications of environmental policy on growth when environmental awareness is endogenously determined by both education and pollution. The government can strengthen its environmental commitment by increasing a tax on pollution and allocate tax revenue between two categories of environmental expenditure: direct one with public pollution abatement and more indirect one through education subsidy. We show that there exists a unique positive BGP, characterized by sustainable development. It can be either in a regime with private environmental maintenance or in a regime with only public environmental maintenance, depending on the design of environmental policy.

When the BGP is with private maintenance, we reveal that the economy may display damped oscillations for low tolerance to pollution. These oscillations are due to the feedback effect of human capital and environmental quality on endogenous green preferences, which shapes the trade-off between private choices. Such complex dynamics makes the welfare vary across generations, and entails intergenerational inequalities. Conversely, when the BGP is in the regime with only public maintenance, there is no more private choices in maintenance and hence damped oscillations do not occur. In addition, we prove that the highest growth rate is achieved in this regime when the share of public spending devoted to education is sufficiently high. In this case, human capital accumulation is favored without damaging the environment.

As a result, we reveal that environmental policy plays a crucial role in avoiding intergenerational inequalities and in improving growth of both human capital and environmental quality. More precisely, we conclude in favor of policy mix and underline that the most favorable policy scheme corresponds to an intermediary allocation of budget between public environmental maintenance and education.

6 Appendix

6.1 Proof of Lemma 1

We study the properties of the dynamical equation $\mathcal{F}(X_t)$, defined on $(-\kappa;+\infty)$.

- When $X_t \in (-\kappa\ ;\ \Lambda)$ the function is given by equation (21 *npm*). We have $\lim_{X_t \to -\kappa} \mathcal{F}(X_t) = \frac{AN(b(1-\tau)+b\tau\sigma-a)-(1-\alpha)\kappa}{\epsilon[\gamma_2\mu A(1-\sigma)\tau]^\mu}$ and

$$\mathcal{F}(0) = \frac{AN(\frac{\beta}{\kappa}\ c_1(1-\tau)b + (\frac{\beta}{\kappa}c_1 + c_2)(b\tau\sigma - a))}{\epsilon\left[A\gamma_2\mu\,(\ c_3(1-\tau) + \varepsilon_2 N(b\sigma\tau - a)) + (1-\sigma)\tau A(\frac{\beta}{\kappa}\ c_1 + (1+\gamma_2\mu)c_3)\right]^\mu \left[\frac{\beta}{\kappa}\ c_1 + (1+\gamma_2\mu)c_3\right]^{1-\mu}}$$

  which is strictly positive under Assumption 1. Finally, with equation (19), $\lim_{X_t \to \Lambda} \mathcal{F}(X_t) = \frac{(1-\alpha)\Lambda + AN(b\sigma\tau - a)}{\epsilon\left[\frac{\gamma_2\mu A(1-\tau)+(1-\sigma)\tau A(1+\gamma_2\mu)}{1+\gamma_2\mu}\right]^\mu}$.

- When $X_t \in [\Lambda\ ;\ +\infty)$, the function is given by equation (21 *pm*). We have $\mathcal{F}(\Lambda) = v > 0$ as $\Lambda \geqslant 0$. Moreover, $\lim_{X_t \to +\infty} \mathcal{F}(X_t) = +\infty$.

As $\lim_{X_t \to \Lambda} \mathcal{F}(X_t) = \mathcal{F}(\Lambda)$, the function is continue on $(\kappa; +\infty)$.

□

6.2 Proof of Proposition 1

pm solution. A BGP in the *pm* regime is characterized by $\bar{X}_{pm} = \mathcal{F}(\bar{X}_{pm})$. Under Assumption 1, $\bar{X}_{pm} \neq 0$. Thus we determine the solutions $\bar{X}_{pm}$ which satisfy $\mathcal{F}(\bar{X}_{pm})/\bar{X}_{pm} = 1$. Using equations (18) and (21 *npm.*), it corresponds to (For the sake of simplicity, subscripts on X are removed):

$$(1-\alpha)\left(\frac{\beta+\eta X}{\kappa+X}c_1 + c_2\right) + \frac{AN\left(\frac{\beta+\eta X}{\kappa+X}c_1(b(1-\tau)+b\tau\sigma-a) - c_2(a-b\tau\sigma)\right)}{X} =$$
$$\epsilon\left[\gamma_2\mu(Ac_3(1-\tau)+\varepsilon_2((1-\alpha)X+AN(b\tau\sigma-a))) + A\tau(1-\sigma)\left(c_1\frac{\beta+\eta X}{\kappa+X} + (1+\gamma_2\mu)c_3\right)\right]^{\mu}\left[\frac{\beta+\eta X}{\kappa+X}c_1 + c_3(1+\gamma_2\mu)\right]^{1-\mu}$$

We define $\mathcal{D}_1(X)$ and $\mathcal{D}_2(X)$, respectively the term on the left and on the right hand side. Under Assumption 1, we have:

- $\lim_{X\to-\kappa} \mathcal{D}_1(X) = \pm\infty$, $\lim_{X\to 0-} \mathcal{D}_1(X) = -\infty$, $\lim_{X\to 0+} \mathcal{D}_1(X) = +\infty$ and $\lim_{X\to+\infty} \mathcal{D}_1(X) = (1-\alpha)(\eta c_1 + c_2) > 0$. Moreover, $\mathcal{D}_1$ is decreasing in X when $X > 0$.
- $\lim_{X\to-\kappa} \mathcal{D}_2(X) = +\infty$, $\lim_{X\to 0} \mathcal{D}_2(X) = \mathcal{C} > 0$, with $\mathcal{C}$ a constant and $\lim_{X\to+\infty} \mathcal{D}_2(X) = +\infty$. Moreover, $\mathcal{D}_2(X) > 0$ and $\mathcal{D}_2$ is decreasing and then increasing with X.

The curves $\mathcal{D}_1(X)$ and $\mathcal{D}_2(X)$ cross at least once in the positive area, what guarantees the existence of a positive solution. From equation (21 *npm.*), we have $d\mathcal{F}(\bar{X}_{pm})/dX < 1$, thus the positive solution $\bar{X}_{pm}$ is unique. A necessary condition to have negative solution ($\bar{X}_{pm} < 0$) is that $\exists\ X \in (-\kappa, 0)$ such that $\mathcal{D}_1(X) > 0$. It corresponds to:

$$\mathcal{T}(X) \equiv (1-\alpha)X(c_1(\beta+\eta X)+c_2(\kappa+X)) + AN[c_1(b(1-\tau)+b\tau\sigma-a)(\beta+\eta X) - (a-b\tau\sigma)c_2(\kappa+X)] < 0$$

Under Assumption 1, $\mathcal{T}(X)$ is increasing in X and $\lim_{X\to-\kappa} \mathcal{T}(X) < 0$ if and only if:

$$-(1-\alpha)\kappa + AN(b(1-\tau) + b\sigma\tau - a) < 0$$

Thus, when $AN(b(1-\tau)+b\sigma\tau-a) > (1-\alpha)\kappa$, $\mathcal{D}_1(X) < 0$ for all $X \in (-\kappa, 0)$ and there is no negative solution. When $AN(b(1-\tau)+b\sigma\tau-a) < (1-\alpha)\kappa$, dynamics may exhibit none, one, or multiple negative equilibria. To ensure the existence of negative solution(s), the following condition has to be satisfied $\mathcal{F}(-\kappa) > -\kappa > X^{\exists}$. From Lemma 1, it corresponds to:

$$\frac{\kappa\varepsilon_2(1-\alpha) - A\varepsilon_1(1-\tau)}{A\varepsilon_2[b(1-\tau)+b\tau\sigma-a]} \leqslant N \leqslant \frac{\kappa(1-\alpha-\epsilon(A(1-\sigma)\tau)^{\mu})}{A[b(1-\tau)+b\tau\sigma-a]}$$

npm solution. The BGP in the *npm* regime is characterized by $\bar{X}_{npm} = \mathcal{F}(\bar{X}_{npm})$. Using (21 *pm*), we obtain:

$$\bar{X}_{npm} = \frac{AN(b\tau\sigma - a)}{\epsilon\left[\frac{A[\gamma_2\mu(1-\tau)+(1-\sigma)\tau(1+\gamma_2\mu)]}{1+\gamma_2\mu}\right]^{\mu} - (1-\alpha)} \qquad (24)$$

According to the condition (17) and under Assumption 1, $\bar{X}_{npm}$ exists only if $b\tau\sigma - a > 0 \Leftrightarrow \sigma > \frac{a}{b\tau}$. To determine the nature of the BGP, we examine the sign of equation (20) when $X_t = \bar{X}_{npm}$. When $\mathcal{P}(\bar{X}_{npm})$ is positive and $\sigma > \frac{a}{b\tau}$ the solution is in the *npm* regime, otherwise the solution is in the *pm* regime. By replacing $\bar{X}_{npm}$ in equation (20), we obtain the function $\mathcal{J}(\tau, \sigma)$, which is increasing in τ and σ. Under Assumption 1, for $\tau = 0$, $\mathcal{J} < 0$ and does no longer depend on σ and for $\tau = 1$, we get $\mathcal{J} > 0 \;\; \forall\, \sigma \in [0, 1]$. We depict a representation of $\mathcal{J}$ at given τ:

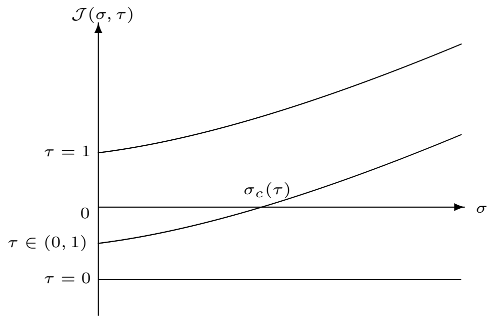


Figure 3: *Function $\mathcal{J}$ at given τ*

We deduce that there exists a $\sigma_c(\tau)$ decreasing in τ such that $\mathcal{J} = 0$, with $\lim_{\tau \to 0} \sigma_c(\tau) = +\infty$ and $\sigma_c(1) < 0$. Thus a minimum level of tax is required to make the npm regime possible. When $\sigma < \sigma_c(\tau)$, we get $\mathcal{P}(\bar{X}_{npm}) < 0$, meaning that the equilibrium is in the *pm* regime. Respectively, when $\sigma \geqslant \sigma_c(\tau)$ we get $\mathcal{P}(\bar{X}_{npm}) \geqslant 0$, and from equations (20) and (24) we have $\bar{X}_{npm} \geqslant \Lambda$ if and only if $\sigma > \frac{a}{b\tau}$. Thus, the BGP is in the *npm* regime when $\sigma > Max\left\{\sigma_c(\tau); \frac{a}{b\tau}\right\} \equiv \hat{\sigma}(\tau)$ and in the *pm* regime when $\sigma < \hat{\sigma}(\tau)$. When $\sigma = \hat{\sigma}(\tau)$, the BGP is in the *npm* regime if $\hat{\sigma}(\tau) = \sigma_c(\tau)$ and in the *pm* regime if $\hat{\sigma}(\tau) = \frac{a}{b\tau}$. Note that under Assumption 1, $\hat{\sigma}(\tau) > \sigma_{Min}(\tau)$ and $\lim_{\tau\to 1} \hat{\sigma}(\tau) < \lim_{\tau \to 1} \sigma_{Max}(\tau)$. □

6.3 Proof of Proposition 2

pm solution. We use the scaling parameter ϵ in order to normalize the steady state $\bar{X}_{pm}$ to one. There is a unique solution ϵ^* such that $\bar{X}_{pm} = 1$ and from equation (21 *npm.*), the expression of the normalization constant is given by:

$$\epsilon^*(\bar{X}_{pm}) \equiv \frac{(1-\alpha)\ \bar{X}_{pm}\ \left[\bar{\gamma_1} c_1 + c_2\right] + AN(\bar{\gamma_1} c_1 b(1-\tau) + (\bar{\gamma_1} c_1 + c_2)(b\tau\sigma - a))}{\left[\gamma_2 \mu A\ c_3(1-\tau) + \gamma_2\mu\varepsilon_2\left[(1-\alpha)\bar{X}_{pm} + AN(b\sigma\tau - a)\right] + (1-\sigma)\tau A(\bar{\gamma_1}\ c_1 + (1+\gamma_2\mu)c_3)\right]^{\mu}\left[\bar{\gamma_1}\ c_1 + (1+\gamma_2\mu)c_3\right]^{1-\mu}}$$

Then, by differentiating equation (21 *npm.*) and analyzing it around the steady state $\bar{X}_{pm} = 1$ and $\epsilon \equiv \epsilon^*(\bar{X}_{pm})$, we obtain:

$$dX_{t+1} = \frac{((1-\alpha)(\bar{\gamma_1}c_1 + c_2 + \bar{\gamma_1}'c_1) + AN\bar{\gamma_1}'c_1(b(1-\tau) + b\tau\sigma - a))\mathcal{B}_1\mathcal{B}_2 - \mathcal{B}_3\left(\mu\mathcal{B}_2(\gamma_2\mu\varepsilon_2(1-\alpha) + (1-\sigma)A\tau\bar{\gamma_1}'c_1) + (1-\mu)\gamma_1' c_1 \mathcal{B}_1\right)}{\mathcal{B}_3\mathcal{B}_1\mathcal{B}_2} dX_t \quad (25)$$

with $\bar{\gamma_1} = \frac{\beta+\eta}{\kappa+1}$, $\bar{\gamma_1}' = (\kappa\eta - \beta)/(1+\kappa)^2$, $\mathcal{B}_1 = \gamma_2\mu A\ c_3(1-\tau) + \gamma_2\mu\varepsilon_2\left[(1-\alpha) + AN(b\sigma\tau - a)\right] + (1-\sigma)\tau A(\bar{\gamma_1}\ c_1 + (1+\gamma_2\mu)c_3)$, $\mathcal{B}_2 = \bar{\gamma_1}\ c_1 + c_3(1+\gamma_2\mu)$ and $\mathcal{B}_3 = (1-\alpha)(\bar{\gamma_1}\ c_1 + c_2) + AN(\bar{\gamma_1}c_1 b(1-$

$\tau) + (\bar{\gamma}_1 c_1 + c_2)(b\tau\sigma - a))$.

From (25), we get $dX_{t+1}/dX_t < 1$. Thus, when $dX_{t+1}/dX_t > 0$, transitional dynamics is monotonous and the BGP equilibrium is locally stable. Using equation (25), we have $dX_{t+1}/dX_t > 0$ if and only if:

$$\begin{aligned} & \mathcal{B}_2(1-\alpha)\left(\bar{\gamma}_1 c_1 + c_2\right)\left[\gamma_2\mu\mathcal{B}_5 + (1-\sigma)\tau A\mathcal{B}_2\right)] \\ - \quad & \mathcal{B}_2(1-\alpha)\gamma_2\mu^2\left[\bar{\gamma}_1 c_1(\mathcal{B}_5 - \varepsilon_1 A(1-\tau)) + c_2\varepsilon_2(1-\alpha + AN(b\tau\sigma - a))\right] \\ + \quad & \bar{\gamma}_1{}' c_1\mathcal{B}_5\left[\gamma_2\mu^2\mathcal{B}_2\mathcal{B}_4 + bN(1+\gamma_2\mu)\left[(1-\mu)\gamma_2\mu\mathcal{B}_5 + (1-\sigma)\tau A\mathcal{B}_2\right]\right] > 0 \end{aligned}$$

with $\mathcal{B}_4 \equiv 1 - \alpha + AN(b(1-\tau) + b\tau\sigma - a)$ and $\mathcal{B}_5 \equiv Ac_3(1-\tau) + \varepsilon_2(1 - \alpha + AN(b\tau\sigma - a))$.

Rewriting this expression, we have $dX_{t+1}/dX_t > 0$ if and only if the following polynomial is positive:

$$\mathcal{R}(\beta) \equiv a_1\beta^3 + a_2\beta^2 + a_3\beta + a_4$$

with $a_4 > 0$ and expressions for a_1, a_2 and a_3 given by:

$$a_1 = \frac{c_1^3}{(1+\kappa)^3}(1-\sigma)(1-\alpha)\tau A > 0$$

$$\begin{aligned} a_2 = \tfrac{c_1^2}{(1+\kappa)^3}(\kappa+1)(1-\alpha)\left[\gamma_2\mu(1-\mu)\mathcal{B}_5 + (1-\sigma)\tau A[2c_3(1+\gamma_2\mu) + c_2] + \gamma_2\mu^2\varepsilon_1 A(1-\tau)\right] \\ + \tfrac{c_1^2}{(1+\kappa)^3}\left[3\eta c_1(1-\alpha)(1-\sigma)\tau A - \mathcal{B}_5\left(\gamma_2\mu^2\mathcal{B}_4 + bAN(1+\gamma_2\mu)\tau(1-\sigma)\right)\right] \end{aligned}$$

$$\begin{aligned} a_3 = \tfrac{c_1^3\eta^2(1-\sigma)(1-\alpha)\tau A}{(1+\kappa)^3} + \tfrac{2\kappa\eta c_1^2}{(1+k)^2}(1-\alpha)\left[\gamma_2\mu(1-\mu)\mathcal{B}_5 + (1-\sigma)\tau A[2c_3(1+\gamma_2\mu) + c_2] + \gamma_2\mu^2\varepsilon_1 A(1-\tau)\right] \\ + \tfrac{c_1(1-\alpha)}{1+\kappa}\left[3(1-\sigma)\tau Ac_2c_3(1+\gamma_2\mu) + (1-\mu)\gamma_2\mu\mathcal{B}_5(c_3(1+\gamma_2\mu) + c_2) + 2\gamma_2\mu^2 A(1-\tau)\varepsilon_1 c_3(1+\gamma_2\mu)\right] \\ + \tfrac{c_1^2\eta(\kappa-1)}{(1+\kappa)^3}\mathcal{B}_5\left[\gamma_2\mu^2\mathcal{B}_4 + bN(1+\gamma_2\mu)(1-\sigma)\tau A\right] \\ - \tfrac{c_1\mathcal{B}_5}{(1+\kappa)^2}\left[\gamma_2\mu^2 c_3(1+\gamma_2\mu)\mathcal{B}_4 + bN(1+\gamma_2\mu)(\gamma_2\mu(1-\mu)\mathcal{B}_5 + (1-\sigma)\tau Ac_3(1+\gamma_2\mu))\right] \end{aligned}$$

There exists a critical level $\bar{\kappa}$ such that $a_2 > 0$ and $a_3 > 0$ for $\kappa > \bar{\kappa}$ with

$$\bar{\kappa} \quad \equiv \quad \frac{\left(\mu^2\gamma_2((1-\alpha)+AN(b(1-\tau)+b\tau\sigma-a))+b\tau(1-\sigma)AN(1+\gamma_2\mu)\right)(Ac_3(1-\tau)+\varepsilon_2(1-\alpha)+\varepsilon_2 AN(b\tau\sigma-a))-3\eta c_1(1-\sigma)\tau A}{(1-\alpha)[\gamma_2\mu\varepsilon_2(1-\mu)(1-\alpha+AN(b\sigma\tau-a))+(1-\sigma)\tau Ac_1(1+\gamma_2\mu)+\gamma_2\mu A(1-\tau)(\varepsilon_1+\varepsilon_2 bN(1-\mu))]} - 1$$

When $\kappa > \bar{\kappa}$, we have that $\mathcal{R}(\beta) > 0$ and dynamics is locally stable and monotonous. Under this level, several scenario may emerge. When $\sigma \in [0,1)$, equation $\mathcal{R}(\beta) = 0$ admits none or two positive solutions. Since $a_3 > 0$, oscillations are observed ($\mathcal{R}(\beta) < 0$) only if the polynomial admits two positive solutions, i.e for intermediary β. For $\tau = 0$ or $\sigma = 1$, $\mathcal{R}(\beta)$ is a polynomial of degree two. In this case, when $\kappa < \bar{\kappa}$ there exists a $\bar{\beta}$ over which the dynamics is oscillatory. We deduce by continuity that for a sufficiently high σ, oscillations occur for intermediary β.

We examine the stability of the equilibrium when dynamic is oscillatory. From equation (25), we have $dX_{t+1}/dX_t > -1$ if and only if:

$$\begin{aligned} & [(1-\alpha)(\bar{\gamma}_1 c_1 + c_2 + \bar{\gamma}_1{}' c_1) + AN\bar{\gamma}_1{}' c_1(b(1-\tau) + b\tau\sigma - a)]\mathcal{B}_1\mathcal{B}_2 + \mathcal{B}_1\mathcal{B}_2\mathcal{B}_3 \\ - \quad & [(1-\mu)c_1\bar{\gamma}_1{}'\mathcal{B}_1 + \mathcal{B}_2\mu\left(\gamma_2\mu\varepsilon_2(1-\alpha) + (1-\sigma)A\tau\bar{\gamma}_1{}' c_1\right)]\mathcal{B}_3 > 0 \end{aligned}$$

Replacing expressions $\mathcal{B}_1$, $\mathcal{B}_1$ and $\mathcal{B}_3$, we finally obtain:

$$\begin{aligned}
& c_3(1+\gamma_2\mu)c_2(1-\alpha)(\mathcal{B}_6+\varepsilon_2 bN\mu)+\gamma_2\mu(\bar{\gamma}_1 c_1)^2(\mathcal{B}_6(1-\alpha)+\mathcal{B}_5\mathcal{B}_4) \\
+ \;& \bar{\gamma}_1 c_1(1+\gamma_2\mu)[(1-\alpha)\varepsilon_1(\mathcal{B}_6+\varepsilon_2 bN\mu)+\gamma_2\mu\varepsilon_1(1-\alpha+AN(b\tau\sigma-a))+c_3(\mathcal{B}_4+\mathcal{B}_6)] \\
+ \;& \gamma_2\mu\bar{\gamma}_1{}' c_1(1+\gamma_2\mu)(\mathcal{B}_4 c_3-(1-\mu)\varepsilon_1(1-\alpha+AN(b\tau\sigma-a))) \\
+ \;& \gamma_2\mu^2\bar{\gamma}_1{}'\bar{\gamma}_1 c_1^2\mathcal{B}_5\mathcal{B}_4+\mathcal{B}_5 c_3(1+\gamma_2\mu)c_2(1-\alpha+AN(b\tau\sigma-a)) \\
+ \;& (1-\sigma)\tau A(\bar{\gamma}_1 c_1+c_3(1+\gamma_2\mu))\gamma_1' c_1 bN(1+\gamma_2\mu)\mathcal{B}_5 \\
+ \;& (1-\sigma)\tau A(\bar{\gamma}_1 c_1+c_3(1+\gamma_2\mu))^2[\bar{\gamma}_1 c_1\mathcal{B}_4+c_2(1-\alpha+AN(b\tau\sigma-a))+(1-\alpha)(\bar{\gamma}_1 c_1+c_2)]
\end{aligned}$$

with $\mathcal{B}_6 = c_2+\varepsilon_2 bN(1+\gamma_2\mu)>0$.

As $-\bar{\gamma}_1{}' < \bar{\gamma}_1$ and $c_3\mathcal{B}_4 > bN\mathcal{B}_5$, we easily see that this term is always positive. The positive BGP equilibrium is always locally stable.

npm solution. The *npm* BGP is obtain from (21 *pm*) and given in Appendix 6.2. We differentiate equation (21 *pm*) and obtain:

$$\frac{d\mathcal{F}(X_t)}{dX_t} = \frac{(1-\alpha)}{\epsilon\left[\frac{A[\gamma_2\mu(1-\tau)+(1-\sigma)\tau(1+\gamma_2\mu)]}{1+\gamma_2\mu}\right]^{\mu}}$$

Under Assumption 1, the slope of $\mathcal{F}(X_t)$ in the *npm* regime is always positive and lower than one, the *npm* BGP is thus monotonously stable.

□

6.4 Proof of Proposition 3

We examine $Sign\left\{\frac{\partial\bar{\kappa}}{\partial\tau}\right\}$ using the expression of $\bar{\kappa}$ given in Appendix 6.3:

$$\begin{aligned}
& Sign\left\{\frac{\partial\bar{\kappa}}{\partial\tau}\right\} = (1-\sigma)bAN(1+\gamma_2\mu(1-\mu))(\mathcal{S}_2-\tau A\varepsilon_2 bN(1-\sigma))(\tau(1-\sigma)\mathcal{S}_3+(1-\tau)\gamma_2\mu A\varepsilon_1) \\
& -(1-\sigma)(b\tau(1-\sigma)AN(1+\gamma_2\mu(1-\mu))+\mathcal{S}_1)(\varepsilon_2 ANb\mathcal{S}_4+\mathcal{S}_3\mathcal{S}_2)-3(1-\sigma)\eta c_1 A\mathcal{S}_4 \\
& +A\varepsilon_1\mu^3(1-\alpha+AN(b-a))(\mathcal{S}_1+b\tau(1-\sigma)AN(1+\gamma_2\mu(1-\mu)))
\end{aligned}$$

with $\mathcal{S}_1 = \mu^2\gamma_2(1-\alpha+AN(b-a))$, $\mathcal{S}_2 = A(1-\tau)\varepsilon_1+\varepsilon_2(AN(b-a)+1-\alpha)$, $\mathcal{S}_3 = A(3\varepsilon_1(1+\gamma_2\mu)+2\varepsilon_2 bN+\gamma_2\mu\varepsilon_2 bN(1+\mu))$ and $\mathcal{S}_4 = \gamma_2\mu(\varepsilon_2(1-\mu)(1-\alpha+AN(b-a))+A\varepsilon_1)$

We can defined $Sign\left\{\frac{\partial\bar{\kappa}}{\partial\tau}\right\}$ as a polynomial of degree three in σ, with $\frac{\partial\bar{\kappa}}{\partial\tau}>0$ when $\sigma=1$ and $\frac{\partial\bar{\kappa}}{\partial\tau}<0$ when $\sigma=0$. Since $Sign\left\{\frac{\partial\bar{\kappa}}{\partial\tau}\right\}$ is increasing in σ for $\sigma\in[0,1]$, there exists a critical value $\tilde{\sigma}\in(0,1)$ such that: for $0<\sigma<\tilde{\sigma}$, $\frac{\partial\bar{\kappa}}{\partial\tau}<0$ and for $\tilde{\sigma}<\sigma<1$, $\frac{\partial\bar{\kappa}}{\partial\tau}>0$. Moreover, under Assumption 1, $\lim_{\tau\to 0}\sigma_{Min}(\tau)<\tilde{\sigma}<\lim_{\tau\to 1}\sigma_{Max}(\tau)$.

□

6.5 Proof of Lemma 2 and Proposition 4

We examine impact of taxation on the BGP growth rate.

***pm* solution.** Using equation (22 *npm*) with $X_t = \bar{X}_{pm}$ we have:

$$\begin{array}{rcl} Sign\left(\frac{\partial g_{pm}}{\partial \tau}\right) & = & \mathcal{V}_2\left((1-\sigma)(\bar{\gamma}_1 c_1 + c_3) - \sigma\gamma_2\mu\varepsilon_1 + \gamma_2\mu\varepsilon_2(1-\alpha)\frac{\partial \bar{X}_{pm}}{\partial \tau}\right) \\ + & & \frac{c_1(\beta-\eta\kappa)}{(\kappa+\bar{X}_{pm})^2}\frac{\partial \bar{X}_{pm}}{\partial \tau}\gamma_2\mu\left(\mathcal{V}_1 - \tau(1-\sigma)A\mathcal{V}_2\right) \end{array}$$

with $\mathcal{V}_1 = \gamma_2\mu A c_3(1-\tau) + \gamma_2\mu\varepsilon_2\left[(1-\alpha)\bar{X}_{pm} + AN(b\sigma\tau - a)\right] + (1-\sigma)\tau A(\bar{\gamma}_1 c_1 + (1+\gamma_2\mu)c_3)$ and $\mathcal{V}_2 = \bar{\gamma}_1 c_1 + (1+\gamma_2\mu)c_3$. From the implicit function theorem and equation (21), we have:

$$\frac{\partial \bar{X}_{pm}}{\partial \tau} = \frac{\mathcal{V}_2\bar{X}_{pm}A[(\bar{\gamma}_1 c_1(\sigma-1)+\sigma c_2)\mathcal{V}_1 N - \mu\mathcal{V}_3(-\sigma\varepsilon_1\gamma_2\mu + (1-\sigma)(\bar{\gamma}_1 c_1 + c_3))]}{\frac{c_1\bar{X}_{pm}(\beta-\eta\kappa)}{(\kappa+\bar{X}_{pm})^2}\big(\mathcal{V}_1\mathcal{V}_2(\bar{X}_{pm}(1-\alpha)+AN(b(1-\tau)+b\tau\sigma-a)) - \mu\mathcal{V}_2\mathcal{V}_3(1-\sigma)\tau A + (1-\mu)\mathcal{V}_1\mathcal{V}_3\big) + \mu\mathcal{V}_2\mathcal{V}_3\bar{X}_{pm}\gamma_2\mu\varepsilon_2(1-\alpha) + AN\mathcal{V}_1\mathcal{V}_2\mathcal{V}_4}$$

with $\mathcal{V}_3 = (1-\alpha)\bar{X}_{pm}\left[\bar{\gamma}_1 c_1 + c_2\right] + AN\left[\bar{\gamma}_1 c_1 b(1-\tau) + (\bar{\gamma}_1 c_1 + c_2)(b\tau\sigma - a)\right]$ and $\mathcal{V}_4 = c_2(b\tau\sigma - a) + \bar{\gamma}_1 c_1 b(1-\tau(1-\sigma))$

Thus, substituting $\frac{\partial \bar{X}_{pm}}{\partial \tau}$ in $Sign\left(\frac{\partial g_{pm}}{\partial \tau}\right)$, we finally obtain:

$$\begin{aligned} Sign\left(\frac{\partial g_{pm}}{\partial \tau}\right) &= \left(\gamma_2\mu\varepsilon_2(1-\alpha)\mathcal{V}_2\bar{X}_{pm}AN + \frac{c_1\bar{X}_{pm}AN(\beta-\eta\kappa)}{(\kappa+\bar{X}_{pm})^2}\left[\mathcal{V}_1 - \tau(1-\sigma)A\mathcal{V}_2\right]\right)(\bar{\gamma}_1 c_1(\sigma-1)+\sigma c_2) \\ &+ \left(\frac{c_1\bar{X}_{pm}(\beta-\eta\kappa)}{(\kappa+\bar{X}_{pm})^2}\left[\mathcal{V}_2(\bar{X}_{pm}(1-\alpha) + AN(b(1-\tau)+b\tau\sigma-a)) + \mathcal{V}_3\right] + \mathcal{V}_2\mathcal{V}_4 AN\right)(-\sigma\varepsilon_1\gamma_2\mu + (1-\sigma)(\bar{\gamma}_1 c_1 + c_3))\mathcal{V}_1 \end{aligned}$$

Under Assumption 1, policy improves the BGP growth rate when the following sufficient condition is satisfied:

$$f_1(\sigma) < \sigma < f_2(\sigma)$$

with $f_1(\sigma) \equiv \frac{\bar{\gamma}_1 c_1}{\bar{\gamma}_1 c_1 + c_2} < 1$ and $f_2(\sigma) \equiv \frac{\bar{\gamma}_1 c_1 + c_3}{\bar{\gamma}_1 c_1 + c_2 + \gamma_2\mu\varepsilon_1} < 1$. These two functions are increasing in $\bar{\gamma}_1$, and as $\frac{\partial \bar{\gamma}_1}{\partial \bar{X}_{pm}} < 0$ and $\frac{\partial \bar{X}_{pm}}{\partial \sigma} > 0$, they are decreasing in σ. As a result, there exists a unique range of value $[\underline{\sigma}(\tau); \overline{\sigma}(\tau)]$ which satisfies this condition. Moreover, under Assumption 1, $\lim_{\tau\to 0}\sigma_{Min}(\tau) < \underline{\sigma}(\tau) < \overline{\sigma}(\tau) < \lim_{\tau\to 1}\sigma_{Max}(\tau)$.

***npm* solution.** We use equation (22 *pm*) with $X_t = \bar{X}_{npm}$ and deduce:

$$Sign\left(\frac{\partial g_c}{\partial \tau}\right) \quad = \quad 1 - \sigma(1+\gamma_2\mu)$$

A tighter tax is growth promoting as long as $\hat{\sigma}(\tau) < \sigma < 1/(1+\gamma_2\mu)$.

Regime switch. We consider the case where an increase in τ leads the economy from a *pm* regime to a *npm* regime. The opposite switch cannot be observed as $\hat{\sigma}(\tau)$ is decreasing in τ. For a given σ, we compare equations (22 *npm*) and (22 *pm*), by considering a higher tax rate in the *npm* regime (τ_N) than in the *pm* one (τ_P). The growth rate in the *pm* regime is higher than in the *npm* if and only if:

$$\begin{aligned} &\gamma_2\mu(1+\gamma_2\mu)\left(c_3 A(\tau_N - \tau_P) + \varepsilon_2((1-\alpha)\bar{X}_{pm} + AN(\sigma b\tau_P - a))\right) \\ &-(1-\sigma)A(\gamma_1 c_1 + (1+\gamma_2\mu)c_3)(\tau_N - \tau_P) - A\gamma_2\mu(1-\tau_N)\gamma_1 c_1 > 0 \end{aligned}$$

This expression is increasing in σ and from (19) is never satisfied when $\sigma = 1/(1+\gamma_2\mu)$. And according to Appendix 6.2, the *npm* regime exists only if $\frac{a}{b} < \sigma$. Thus, for a given $\sigma \in (a/b\,; 1/(1+\gamma_2\mu))$, the growth rate in the *npm* regime is higher than in the *pm* one. Moreover, under Assumption 1, $\sigma_{Min}(\tau) < 1/(1+\gamma_2\mu) < \sigma_{Max}(\tau)$.

□

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