Optimum Growth and Carbon Policies with Lags in the Climate System

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Abstract

We study the effects of greenhouse gas emissions on optimum growth and climate policy by using an endogenous growth model with polluting non-renewable resources. Climate change harms the capital stock. Our main contribution is to introduce and extensively explore the naturally determined time lag between greenhouse gas emission and the damages due to climate change, which proves to be crucial for the transition of the economy towards its steady state. The social optimum and the optimal abatement policies are fully characterized. The inclusion of a green technology delays optimal resource extraction. The optimal tax rate on emissions is proportional to output. Poor understanding of the emissions diffusion process leads to suboptimal carbon taxes and suboptimal growth and resource extraction.

Keywords: Non-Renewable Resource Extraction; Climate Policy; Optimum Growth; Pollution Diffusion lag.

JEL-Classification: Q54; O11; Q52; Q32

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1 Introduction

Climate change has certain characteristics which impede the implementation of optimal environmental policies. First, it has a global dimension, necessitating difficult international negotiations and agreements. Second, mitigation policies create substantial and unevenly distributed economic costs and benefits across different countries. Third, climate change requires consideration of a very long time horizon. This poses a major challenge for political decision making which is usually rather myopic; in the past most environmental policies were only implemented after major environmental damages had been publicly observed, creating some political necessity to act.[1] But with climate change, damages will only be fully visible after several decades, because greenhouse gas emissions cause economic damages only after a major time lag.[2] The existence and nature of this delay in the natural system has major implications for optimum growth and carbon policies, which we study in this paper.

The present paper develops a theoretical model of a growing economy that is harmed by climate change. It contributes to the theoretical literature in the following ways: first, the paper introduces a well-specified time lag between emissions from non-renewable resources and the harmful pollution they cause. With our specification, an increase in the stock of greenhouse gases only gradually increases the stock of harmful pollution, thus allowing for a diffusion process between the two stocks. To the best of our knowledge, the paper is the first to derive the impact of this time lag in pollution dissemination in terms of closed-form solutions. Second, we propose a simple dynamic economic climate model based on the seminal contribution of Rebelo (1991) in order to study the effects of climate change. The model incorporates relevant features such as carbon emissions being caused by non-renewable resources, climate change affecting capital depreciation, and endogenous growth relying on capital investment. Third, we derive closed-from solutions for both the social optimum and climate policies deviating from the optimum. To study the involved time lags thoroughly is in our view a crucial contribution to designing optimum climate policies. Finally, we extend the analysis to the case where a clean energy input is available and show that the basic results continue to hold. Clean energy inputs are shown to delay resource extraction, shifting pollution to the future. Thus, by comparing the social optimum to different forms

1 Examples are the Montreal Protocol on the ozone layer or the ban of asbestos.

2 In the Stern Review is stated "climate models project that the world is committed to a further warming...over several decades due to past emissions.", Stern (2007), p.15.

of climate policies with and without the existence of clean energies the paper contributes to the theoretical literature on growth and climate change. By relating current policies to actual observation of climate damages it offers an explanation for insufficient efforts in current environmental policy.

The paper is related to existing literature in different fields. Time lags in the climate system are usually implemented in integrated climate assessment models. Prominent examples are Nordhaus (1992), Nordhaus (2011), and Rezai et al. (2012), who calibrate a Ramsey growth model to show a significant Pareto-improvement due to climate mitigation investment. Another related paper is Golosov et al. (2014), which introduces non-renewable resources as in our model but abstract from capital stocks, which are crucial for our approach to capture both endogenous growth and climate damage. In line with their purpose and due to their complexity these models do not provide closed-form solutions.

As regards the theoretical models, Withagen (1994) shows that the introduction of pollution with non-renewable resource use delays optimum resource extraction, which will also be the case in our model. Tahvonen (1997) additionally allows for a non-polluting backstop technology and defines different switching regimes between non-renewable resources and the backstop, which depend on initial pollution and the price of non-renewable resources and the backstop. Hoel and Kverndokk (1996) abstract from finiteness of non-renewable resources by focusing on the economic recoverability of the resource stock. They also note that in the presence of greenhouse effects it will be optimal to slow down extraction and spread it over a longer period. These models abstract from capital accumulation, which is crucial for growth, and capital destruction due to climate change, which represents climate damages in a more realistic way.

The impact of environmental pollution and policy on economic growth is studied by Bovenberg and Smulders (1995). Michel and Rotillon (1995) derive the optimum tax policy in a linear growth model with disutility of pollution stocks [3]. Grimaud and Rouge (2014) analyze how the availability of an abatement technology affects optimal climate policies using an endogenous growth model based on the expansion-in-varieties framework and show that when such a technology is available the optimal carbon tax is uniquely determined. In a

[3] For a survey of the literature on the relationship between environmental pollution and growth, see Brock and Taylor (2005)

Ramsey growth model, van der Ploeg and Withagen (2010) analyze optimal climate policy depending on the social cost of oil and renewable resources. Ikefuji and Horii (2012) develop a model with capital destruction due to climate change and conclude that growth is sustainable only if the tax rate on the polluting input increases over time. Contrary to our model they abstract from resource finiteness and pollution stock. Using an endogenous growth model, Bretschger and Valente (2011) show that less developed countries are likely to be hurt more than developed ones, by greenhouse gas emissions inducing negative growth deficits and possible unsustainability traps. Finally, Bretschger and Suphaphiphat (2013) find that climate mitigation policies by the North are more effective than additional development aid to the South. Most theoretical models on climate change have sidestepped time lags in the climate system. An important exception is the contribution of Gerlagh and Liski (2012); the main differences to our approach are that they neither introduce capital nor non-renewable resource stocks, as we do in the present approach.

The remainder of the paper is organized as follows. Section 2 presents the basic model. In Section 3 we characterize optimum growth. Section 4 introduces clean energy inputs. In section 5 we analyze different tax rates, present empirical evidence, and discuss the implication on tax policies. Section 6 concludes.

2 The basic model

The model framework used in the paper is based on the endogenous growth approach of Rebelo (1991). We regard an economy with two representative production sectors, the consumption goods sector (C) and the investment goods sector (I), which share the capital (K) that is available to the economy for production; both sectors exhibit constant returns to scale. [4] The consumption sector combines a fraction ϵ of total capital with non-renewable resources (R), using a Cobb-Douglas production function, so that the output of consumer goods reads

$$C_t = F(\epsilon_t K_t, R_t) = A(\epsilon_t K_t)^{\alpha} R_t^{1-\alpha} \tag{1}$$

where $A > 0$ is a constant. The capital sector uses the remaining capital stock to produce investment goods (I) according to

$$I_t = B(1-\epsilon_t)K_t. \tag{2}$$

[4] For simplicity, we do not introduce labor as a separate input but interpret capital K in a broad sense, i.e. to include also human capital.

with $B > 0$ being a constant [5]. The stock of harmful pollution (P) in the atmosphere acts negatively on capital accumulation by deteriorating the existing capital stock. We denote by $D(P_t) = \chi P_t$ the capital depreciation rate as a function of harmful pollution stock; χ is depreciation intensity, assumed to be a small number so that $D(P_t) \ \epsilon \ (0, 1]$. We then get

$$\dot{K}_t = I_t - D(P_t)K_t, \qquad D(P_t) = \chi P_t. \tag{3}$$

The use of non-renewable resources (R) depletes the existing resource stock (S) by the usual constraint

$$\dot{S}_t = -R_t. \tag{4}$$

Non-renewable resource use, with a carbon intensity ϕ, increases the concentration of greenhouse gases in the atmosphere, defined as the stock of accumulated emissions (E), according to

$$\dot{E}_t = \phi R_t. \tag{5}$$

We do not include a pollution decay parameter, because decay of carbon in the atmosphere is very slow and does not obey a simple functional form (in section 5.3 we provide a short discussion on how would the optimal tax look like in the presence of pollution decay as usually introduced in the literature). Unlike the vast majority of the theoretical literature, which assumes an immediate response of environmental damages to greenhouse gas emissions, we differentiate between the stock of accumulated emissions (E) i.e. the concentration of greenhouse gases in the atmosphere, and the harming pollution stock (P) in order to introduce a diffusion process between the two.

Pollution diffusion lags are highly relevant for the atmospheric system[6] and the transition of the economy towards its steady state, which we derive in the present paper. We explicitly accommodate emission dissemination by assuming that resource use creates emissions that

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[5] We assume that production is performed by a unit mass of firms, $j \epsilon [0, 1]$, in competition. The consumption good sector produces according to $c_j = Ak_{c,j}^{\alpha} R_j^{1-\alpha}$, where $k_{c,j}$ corresponds to a capital unit from the overall physical capital devoted to the consumption good sector, $K_c = \epsilon K$, and R_j is the demand for non-renewable resources by firm j. Constant returns to scale in production implies the same factor input ratios so that in aggregate, with $C = \int c_j dj$ and $R = \int R_j dj$, we get (1). Same logic applies to the investment good sector giving (2).

[6] The IPCC (2007) report points out the complexity of the carbon cycle. Organic and inorganic processes (referred to as organic or inorganic pumps) maintain a vertical gradient of CO2 between deep oceans, the sea surface and the atmosphere introducing, in this way, a time lag between emissions and pollution accumulation.

contribute to the harmful pollution stock only with a, possibly considerable, time lag, i.e. we study in continuous time the effect of a prolonged period of emissions dissemination. Notably, we specify harmful pollution at time t as

$$P_t = \int_{-\infty}^{t} \left(1 - e^{-\kappa(t-\upsilon)}\right) \dot{E}_\upsilon d\upsilon \tag{6}$$

so that an increase in emission stock ($\dot{E}$) only gradually increases the stock of harmful pollution. Specifically, equation (6) states that P depends on all past emissions and that an emission unit that happened s time units ago contributes by $1 - e^{-\kappa s}$ emission units to the current stock of harmful pollution[7]. Parameter κ measures the speed of dissemination or adjustment speed of the harmful pollution to the stock of emissions. The higher κ is, the shorter the time lag becomes; $\kappa \to \infty$ representing the limiting case of instantaneous diffusion. We set $\kappa > 0$ since polluting emissions are inherently essential for harmful pollution. By differentiating (6) using the Leibniz integral rule one obtains the law of motion for pollution accumulation, which depends on κ and E

$$\dot{P}_t = \kappa(E_t - P_t). \tag{7}$$

According to (4) and (5) the emissions accumulation can be rewritten as $E_t = E_0 + \phi(S_0 - S_t)$ where S_0 and E_0 are the initial non-renewable resource stock and emissions stock. Pollution accumulation follows from (7). We posit, without loss of generality, that the initial harmful pollution stock P_0 is different from the initial accumulated emissions so that

$$\dot{P}_t = \kappa[E_0 + \phi(S_0 - S_t) - P_t]. \tag{8}$$

Finally we define households. The representative agent owns the physical capital in the economy and resource extraction rights. Individuals have logarithmic preferences and derive utility (U) from current and future consumption (C) according to

$$U = \int_0^\infty \ln(C_t) e^{-\rho t} dt \tag{9}$$

where ρ is the rate of time preference.

[7] An equivalent specification that relates the two stocks, the accumulated emissions stock - E, and the stock of harmful pollution - P, would be $P_t = \int_{-\infty}^{t} \kappa e^{-\kappa(t-\upsilon)} E_\upsilon d\upsilon$.

3 Optimum growth

The social planner maximizes (9) subject to (1), (3), (4), (8). To simplify notation we define an emissions intensity parameter as a function of κ, $\zeta(\kappa)$

$$\zeta \equiv \frac{\phi\chi\alpha}{\rho(\frac{\rho}{\kappa}+1)} \tag{10}$$

while $g_{z,t} \equiv \hat{z}_t \equiv \frac{\dot{z}_t}{z_t}$ denotes the growth rate of z_t. Appendix A shows that in equilibrium the capital share between the two sectors is constant

$$\epsilon = \frac{\rho}{B} \tag{11}$$

and the growth rates read

$$g_{K,t} = B - \rho - \chi P_t, \qquad g_t = g_{C,t} = \alpha g_{K,t} + (1-\alpha) g_{R,t}. \tag{12}$$

We then state

Proposition 1 *In a social optimum solution*

(i) capital share ϵ immediately jumps to its steady state,

(ii) capital growth is negatively proportional to the level of harming pollution.

Proof: Conclusions (i)-(ii) directly follow from (11) and (12).

The proposition reveals that the macroeconomic part of the model remains very tractable, despite the complexity added by the emission dissemination mechanism. Furthermore, we derive in Appendix A that the resource depletion growth rate and the level of extraction are given by

$$g_{R,t} = \frac{-\rho}{1 + e^{-\rho t}\left(e^{\frac{S_0\zeta}{1-\alpha}} - 1\right)}, \qquad R_t = \frac{(1-\alpha)\frac{\rho}{\zeta}}{1 + e^{\rho t}\left(e^{\frac{S_0\zeta}{1-\alpha}} - 1\right)^{-1}} \tag{13}$$

with $\lim_{t\to\infty} g_R = -\rho$. Accumulated emissions follow

$$E_t = E_0 + (1-\alpha)\frac{\phi\rho}{\zeta}t - (1-\alpha)\frac{\phi}{\zeta}\ln\left[1 + e^{-\frac{S_0\zeta}{1-\alpha}}(e^{\rho t} - 1)\right], \tag{14}$$

and the accumulation of harmful pollution is given by

$$P_t = E_t + e^{-\kappa t}(P_0 - E_0) + (1-\alpha)\frac{\phi\rho}{\zeta\kappa}\left(e^{-\kappa t}\Psi_t - \Omega_t\right), \tag{15}$$

with

$$\Psi_t = \left(1 - e^{-\frac{S_0\zeta}{1-\alpha}}\right) \sum_{n=0}^{\infty} n! \frac{\Gamma\left(\frac{\kappa}{\rho}+1\right)}{\Gamma\left(\frac{\kappa}{\rho}+1+n\right)} \frac{1}{e^{n\frac{S_0\zeta}{1-\alpha}}}, \tag{16}$$

$$\Omega_t = \frac{1}{1 - e^{\rho t}\left(1 - e^{\frac{S_0\zeta}{1-\alpha}}\right)^{-1}} \sum_{n=0}^{\infty} n! \frac{\Gamma\left(\frac{\kappa}{\rho}+1\right)}{\Gamma\left(\frac{\kappa}{\rho}+1+n\right)} \frac{1}{\left(1 - e^{-\rho t}\left(1 - e^{\frac{S_0\zeta}{1-\alpha}}\right)\right)^{n}}. \tag{17}$$

Γ refers to the gamma function $\Gamma(n) = (n-1)!$ (Proof see Appendix A). This leads to our next proposition.

Proposition 2 *With a time lag in emission diffusion ($0 < \kappa < \infty$) in a social optimum solution*

(i) resource extraction is continuously decreasing, the extraction growth rate asymptotically converges to a negative constant ($-\rho$),

(ii) rising speed of emission diffusion (increasing κ) results in a steeper resource extraction growth profile,

(iii) economic growth (g) is highest at the beginning, converging asymptotically to a constant,

(iv) decreasing speed of emission diffusion (decreasing κ) flattens the growth profile of the economy,

(v) initial economic growth (g_0) is increasing in emission diffusion speed (κ), for $\kappa \to \infty$ it converges to its highest optimum value.

Proof: See Appendix A.

The proposition summarizes the impact of resource extraction, emissions and pollution stock on economic development. The intuition behind the resource extraction profile in (i) and (ii) directly follows from equation (13). If the impact of current emissions were immediate ($\kappa \to \infty$), the social planner would have to stretch polluting resource extraction so that the economy could accumulate sufficient capital in order to substitute away early from the harmful non-renewable resource. Delaying resource extraction in the presence of an environmental externality when emissions instantaneously diffuse into the pollution stock is a well-known result in literature, see Withagen (1994) and Hoel and Kverndokk (1996). We add to the existing literature by showing how the resource extraction path depends on the time lag between emissions and pollution impact and that when this is long (small κ), faster depletion is expected. In this case of small κ, since the harmful effects only occur at a later stage, resource use at an early stage is beneficial for the economy, only becoming increasingly harmful later on. Figure 1 provides a graphical representation of the optimal path of resource depletion growth rate (g_R) in the three dimensions as a function of the emissions dissemination speed (κ) and time (t) on the left hand side and the development of resource depletion growth rate in time for three different values of dissemination speed (κ) on the right hand side. A small κ (κ_1), implying a slow speed of emissions dissemination, corresponds to the case of the fastest resource depletion among these three cases, closer to $-\rho$. The opposite applies for κ_3 (high κ).

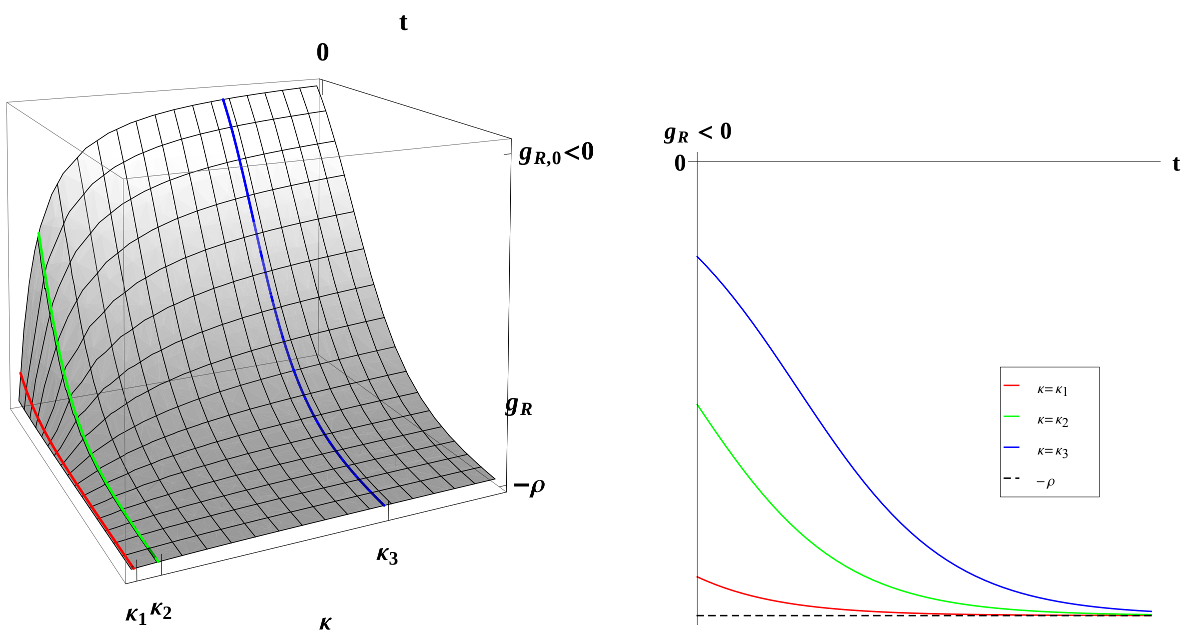

Figure 1: Resource extraction growth rate for different values of κ ($\kappa_1 < \kappa_2 < \kappa_3$)

The intuition regarding parts (iii)-(v) of the proposition follows directly from equations (12) and (14)-(17). The faster the response of the harmful pollution stock to current emissions is (high κ), the higher the initial growth and the steeper the growth profile of the economy becomes. Figure 2 illustrates the optimal path of the consumption growth rate (g) in the three dimensions as a function of the dissemination speed (κ), and time (t) on the left hand side and the development of this growth rate in time for three different values of dissemination speed (κ) on the right hand side. During transition, pollution asymptotically approaches its highest value, but the economy can increasingly accumulate capital so that the marginal pollution effect becomes smaller. Furthermore, since the discounted stream of consumption over the whole time horizon must be constant regardless of κ, the steeper the growth profile is, the higher is the initial point of the economy.

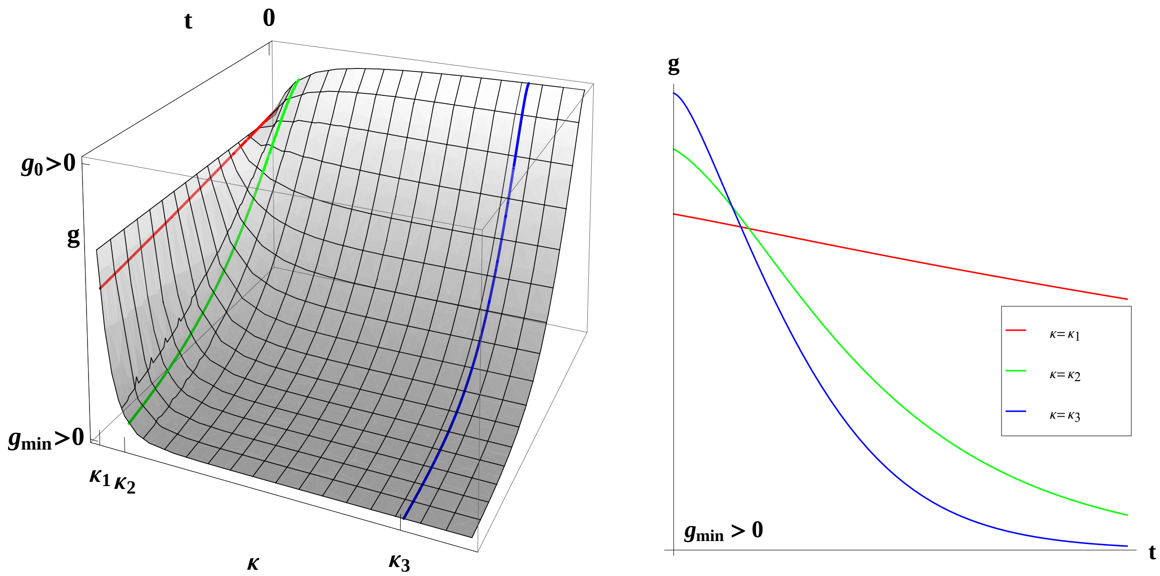

Figure 2: Consumption growth rate for different values of κ ($\kappa_1 < \kappa_2 < \kappa_3$)

It turns out that dissemination speed κ is a crucial parameter for the results, i.e. the transition of the economy to the steady state. The function $\zeta(\kappa)$ is concave, increasing, and continuous in κ for the relevant area of $\kappa \geq 0$, see Figure 3. For $\kappa = 0$, implying $\zeta = 0$, where emissions do not contribute to the harmful pollution stock, the economy jumps immediately to its steady state. Equation (12) gives the growth rate of the economy, $g_{\kappa=0}$ by setting $g_{K,\kappa=0} = B - \rho - \chi P_0$ and $g_{R,\kappa=0} = -\rho$. Sustainable growth depends then on $\alpha B - \alpha\chi P_0 - \rho > 0$. The limiting case of immediate diffusion of emissions to the stock of pollution follows for $\kappa \to \infty$. In that case $\zeta \to \frac{\phi\chi\alpha}{\rho}$ and, as we can see from (6), $lim_{\kappa\to\infty} P_t = E_t$. The growth rate of the economy following (12) starts from a higher point converging asymptotically to its lower steady state at the highest speed where sustainable growth depends on $\alpha B - \alpha\chi(E_0 + \phi S_0) - \rho > 0$. The above can be confirmed by using figure 2 where we illustrate the graphs of the growth rate g for different values of κ. Slow emissions diffusion process and flatter profile corresponds to κ_1 (small κ). The opposite applies for κ_3 (high κ).

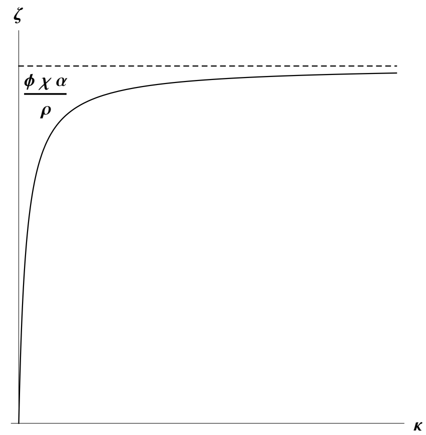

Figure 3: The emissions intensity function $\zeta(\kappa)$

4 Clean energy

In this section we introduce clean energy infrastructure (M) as an essential input into the production of the consumption good. Specifically, M represents the stock of clean energy capital available to the economy, such as windmills and photovoltaic installations. Clean energy stock is accumulated by investing a share ϵ_M of total capital, according to

$$I_{M,t} = \tilde{B}(\epsilon_{M,t} - \theta)K_t, \tag{18}$$

with $\tilde{B}$ being a positive constant. In an economy where green infrastructure is available we define the parameter θ, with $0 < \theta < \epsilon_{M,t} \leq 1$, as the premium to pay for the clean technology.[8] We thus assume that clean energy infrastructure has a higher cost than regular capital. The economy now consists of the consumption sector and two capital accumulating sectors (physical capital K and clean energy infrastructure M). For simplicity, pollution affects both stocks in the same way so that clean energy infrastructure accumulation reads

$$\dot{M}_t = I_{M,t} - D(P_t)M_t, \qquad D(P_t) = \chi P_t. \tag{19}$$

The consumption good sector uses M in addition to K and R in a Cobb-Douglas fashion, keeping the assumption of constant returns to scale. We assume that both energy types are essential inputs for the production of the consumption good. This reflects the (realistic) prediction that some sectors and economic activities like long range transportation will continue

[8] The variable ϵ_M will be determined endogenously. We then have to restrict the value of θ so that the inequality holds. Using (22) we identify $\theta \, \epsilon \left(0, 1 - \frac{\gamma}{\alpha+\gamma}\frac{\rho}{B}\right)$.

to rely on non-renewable energy resources in the future. Moreover, we assume that the economy is not able to produce without renewable energies ensuring that this input will be always needed in the future. In order to be able to get a direct comparison with the results of the previous section, we restrict the expenditure shares for non-renewable resources and clean infrastructure to add up to β, the expenditure share for energy use in the previous section[9]. The variable ϵ_C denotes the capital share which is allocated to the consumption sector, so that

$$C_t = F(\epsilon_{C,t}K_t, R_t, M_t) = A(\epsilon_{C,t}K_t)^{\alpha} R^{\beta'} M^{\gamma}, \quad \alpha + \beta' + \gamma = 1, \quad \beta' + \gamma = \beta. \tag{20}$$

Investment into new capital units follows the same specification as in (3) with $\epsilon_{K,t}$ being the share of the capital stock invested into physical capital accumulation

$$\dot{K}_t = B\epsilon_{K,t}K_t - D(P_t)K_t, \qquad D(P_t) = \chi P_t. \tag{21}$$

Capital shares add up to unity, i.e. $\epsilon_C + \epsilon_K + \epsilon_M = 1$. We show in Appendix B that the allocation of physical capital between the different sectors is constant, according to

$$\epsilon_C = \frac{\frac{\rho}{B}}{1 + \frac{\gamma}{\alpha}\left(1 - \frac{\rho}{B(1-\theta)}\right)}, \ \epsilon_K = 1 - \frac{\rho}{B} - \theta, \ \epsilon_M = \frac{\frac{\rho}{B}}{1 + \left[\frac{\gamma}{\alpha}\left(1 - \frac{\rho}{B(1-\theta)}\right)\right]^{-1}} + \theta. \tag{22}$$

The growth rates then read

$$g_{K,t} = g_{M,t} = B(1-\theta) - \rho - \chi P_t, \qquad g_t = (\alpha + \gamma) g_{K,t} + (\beta - \gamma) g_{R,t}. \tag{23}$$

Proposition 3 *When allowing for a clean energy infrastructure as an essential input to the production of the consumption good:*

(i) capital shares ϵ_C, ϵ_K and ϵ_M jump immediately to their steady state,

(ii) physical capital and clean infrastructure grow in parallel, i.e. $g_K = g_M$,

(iii) economic growth is higher, the smaller is the premium to pay for green infrastructure and the higher is the expenditure share for it in comparison to non-renewable resources.

Proof: Conclusion (i) directly follows from (22). Conclusion (ii) is proven in Appendix B but the intuition behind that is straightforward and follows from (19) since pollution acts on

[9] In a world without clean energy infrastructure, when $\gamma = 0$, we restrict $\theta = 0$.

$\dot{K}$ and $\dot{M}$ in the same way. Conclusion (iii) follows from (23).

Furthermore, we derive in Appendix B that with $\tilde{\zeta} \equiv \frac{\phi\chi(\alpha+\gamma)}{\rho(\frac{\rho}{\kappa}+1)}$ resource depletion growth rate and level of extraction are given by

$$g_{R,t} = \frac{-\rho}{1+e^{-\rho t}\left(e^{\frac{S_0\tilde{\zeta}}{\beta-\gamma}}-1\right)}, \qquad R_t = \frac{(\beta-\gamma)\frac{\rho}{\tilde{\zeta}}}{1+e^{\rho t}\left(e^{\frac{S_0\tilde{\zeta}}{\beta-\gamma}}-1\right)^{-1}} \tag{24}$$

with $\lim_{t\to\infty} g_R = -\rho$. GHG concentration follows

$$E_t = E_0 + (\beta-\gamma)\frac{\phi\rho}{\tilde{\zeta}}t - (\beta-\gamma)\frac{\phi}{\tilde{\zeta}}\ln\left[1+e^{-\frac{S_0\tilde{\zeta}}{\beta-\gamma}}(e^{\rho t}-1)\right], \tag{25}$$

and the accumulation of harmful pollution is given by

$$P_t = E_t + e^{-\kappa t}(P_0 - E_0) + (\beta-\gamma)\frac{\phi\rho}{\tilde{\zeta}\kappa}\left(e^{-\kappa t}\Psi_t - \Omega_t\right), \tag{26}$$

with

$$\Psi_t = \left(1-e^{-\frac{S_0\tilde{\zeta}}{\beta-\gamma}}\right)\sum_{n=0}^{\infty} n!\frac{\Gamma\left(\frac{\kappa}{\rho}+1\right)}{\Gamma\left(\frac{\kappa}{\rho}+1+n\right)}\frac{1}{e^{n\frac{S_0\tilde{\zeta}}{\beta-\gamma}}} \tag{27}$$

$$\Omega_t = \frac{1}{1-e^{\rho t}\left(1-e^{\frac{S_0\tilde{\zeta}}{\beta-\gamma}}\right)^{-1}}\sum_{n=0}^{\infty} n!\frac{\Gamma\left(\frac{\kappa}{\rho}+1\right)}{\Gamma\left(\frac{\kappa}{\rho}+1+n\right)}\frac{1}{\left(1-e^{-\rho t}\left(1-e^{\frac{S_0\tilde{\zeta}}{\beta-\gamma}}\right)\right)^n}. \tag{28}$$

We are now ready to prove

Proposition 4 *Given the availability of a clean energy input (M):*

(i) resource extraction is slower at each instance of time, the higher the share (γ) of the clean input to the production of the consumption good is; extraction growth asymptotically converges to $-\rho$,

(ii) along the transition to the steady state the stocks of emissions and harmful pollution are lower than without clean input. They asymptotically converge to the same value as in the case without M.

(iii) at each point of time consumption growth rate (g) is always higher than in the case without M.

Proof: To prove (i) we have to compare g_R from (24) with the one from (13) (which we denote as g_R^M). Since $\beta \geq \gamma$, it holds that $g_{R,0}^M \geq g_{R,0}$ and that both g_R, and g_R^M are strictly decreasing while their paths cannot cross for $\gamma > 0$, so that $g_R^M \geq g_R$ at each time instance (note that $g_R, g_R^M < 0$). They both converge to $-\rho$ as $t \to \infty$. Conclusion (ii) directly follows as a consequence of (i). By comparing (23) with (12) using (i) and (ii) for the relevant range of $\theta \; \epsilon \left(0, 1 - \frac{\gamma}{\alpha+\gamma} \frac{\rho}{B}\right)$ (see footnote 8) we get (iii).

According to this proposition economic growth is higher than without the clean energy infrastructure at any time instance, since higher γ slows resource extraction, stretching it to the future. As a consequence of that, maximum pollution occurs further in the future. By that time the economy has accumulated enough capital, and the marginal damages of pollution to the accumulation of the capital stocks are small. We conclude that the social optimum with the clean energy input has new beneficial features for the economy while the impact of κ remains the same as in the previous section without M.

5 Climate policy

In this section we model climate policy by introducing a polluting emissions tax in decentralized equilibrium. Since harmful pollution occurs with a lag, the effects of emissions can only be observed with a delay. Therefore, if myopically determined, carbon tax rates are likely to depend on current observations or on wrong perceptions of the delay. Then, the tax rate on polluting emissions would be smaller the lower the perceived speed of emissions diffusion to the destructive pollution stock is. Accordingly, one would then expect that the parameter κ be of great importance for current climate policies. To derive the exact expression of the Pigouvian tax which restores the first-best allocation, we shall compare the results of the current section to the results of the social optimum in section 3. The decentralized equilibrium is not studied for the case of clean energy infrastructure but the solution for this case is straightforward once following the same procedure and comparing to section 4.

5.1 Decentralized optimization

The representative firm of the economy produces the consumption and the investment good according to (1) and (2), operating under perfect competition. To maximize profits it operates under the no-arbitrage condition that employing a marginal unit of capital to produce either good should yield the same return

$$p_t B = \alpha(\epsilon K_t)^{\alpha-1} R_t^{1-\alpha} \tag{29}$$

where $p = \frac{p_I}{p_C}$ is the relative price of investment in terms of consumption, i.e. the real price of investment into new capital units. Furthermore, given that p_t is not constant, capital denominated loans will differ from consumption denominated loans, thereby accounting for the relative price changes. Accordingly, the marginal productivity of capital in the investment good sector should equal the capital denominated interest rate plus the capital depreciation

$$B = r_{K,t} + D(P_t). \tag{30}$$

In equilibrium, marginal productivity of non-renewable resource use equals its unit cost, i.e.

$$(1-\alpha)\frac{C_t}{R_t} = p_{R,t} + \phi\tau_t \tag{31}$$

with p_R denoting the price of the non-renewable resource and τ the per-unit emissions tax in real terms (consumption denominated).

Households own capital and non-renewable resources and rent them to firms, accounting for relative prices. They wish to maximize (9) according to (4) and their wealth evolution is[10]

$$\dot{K} = r_{K,t}K_t + \frac{p_{R,t}}{p_t}R_t - \frac{1}{p_t}C_t + T_t. \tag{32}$$

T_t are lump-sum capital denominated tax revenue transfers. The first order conditions of this maximization give the usual Keynes-Ramsey rule $g = g_C = r_C - \rho$. The variable $g = \widehat{C}$ denotes the growth rate of consumption with r_C being the gross of depreciation (consumption denominated) interest rate. Since households care for relative price changes we have $r_C = r_K + \widehat{p}$ so that

$$g_t = r_{K,t} + \widehat{p}_t - \rho. \tag{33}$$

Controlling for non-renewable resource extraction gives the Hotelling rule for the evolution of the price of the non-renewable resource as

$$\widehat{p}_{R,t} = r_{K,t} + \widehat{p}_t. \tag{34}$$

By combining (33) with (29) and (30) and using the fact that from (1) $g = \alpha g_K + (1-\alpha)g_R$, since ϵ will be constant [11], we get the expression for $g = g_C$

$$g_t = \alpha(B - \rho - \chi P_t) + (1-\alpha)g_{R,t}. \tag{35}$$

[10] Nominal GDP in investment good terms reads $\frac{1}{p}C + I = (r - D(P))K + \frac{p_R}{p}R + T$. Using (3) and by denoting $r_K = r - D(P)$ we get (32).

[11] Since the representative firm operates competitively under the no-arbitrage condition (29) without accounting for the overall capital destruction due to pollution, capital allocation between the two sectors jumps immediately to its steady state.

5.2 Impact of taxes

We now derive the solutions with a flexible pollution tax and compare them to the optimum allocation of the social planner. As in the basic model we employ $\zeta = \frac{\phi\chi\alpha}{\rho(\frac{\rho}{\kappa}+1)}$ to simplify exposition; the superscript "D" denotes the decentralized solution and "o" the optimum level. As shown in Appendix C an optimum emissions tax rate that restores the first best allocation reads

$$\tau_t^o = \frac{\zeta}{\phi\rho}C_t. \tag{36}$$

In the presence of a carbon tax, resource extraction is delayed. Zero tax would lead to the fastest extraction with $g_{R,t} = -\rho$, as can be seen from

$$R_t^D = \begin{cases} \frac{(1-\alpha)\rho/\zeta}{\frac{\tau_t}{C_t}\frac{\phi\rho}{\zeta}\left[1+e^{\rho t}\left(e^{\frac{S_0\zeta}{1-\alpha}\frac{\tau_t}{C_t}\frac{\phi\rho}{\zeta}}-1\right)^{-1}\right]} & \text{if } \tau \neq 0 \\ S_0\rho e^{-\rho t} & \text{if } \tau = 0 \end{cases}, \tag{37}$$

$$g_{R,t}^D = \begin{cases} \frac{-\rho}{1+e^{-\rho t}\left(e^{\frac{S_0\zeta}{1-\alpha}\frac{\tau_t}{C_t}\frac{\phi\rho}{\zeta}}-1\right)} & \text{if } \tau \neq 0 \\ -\rho & \text{if } \tau = 0 \end{cases}. \tag{38}$$

The stocks of emissions and harmful pollution are given by[12]

$$E_t^D = \begin{cases} E_0 + (1-\alpha)\frac{\frac{\phi\rho}{\zeta}}{\frac{\tau_t}{C_t}\frac{\phi\rho}{\zeta}}t - (1-\alpha)\frac{\frac{\phi}{\zeta}}{\frac{\tau_t}{C_t}\frac{\phi\rho}{\zeta}}\ln\left[1+e^{-\frac{S_0\zeta}{1-\alpha}\frac{\tau_t}{C_t}\frac{\phi\rho}{\zeta}}\left(e^{\rho t}-1\right)\right] & \text{if } \tau \neq 0 \\ E_0 + \left(1-e^{-\rho t}\right)\phi S_0 & \text{if } \tau = 0 \end{cases} \tag{39}$$

$$P_t^D = \begin{cases} E_t^D + e^{-\kappa t}(P_0 - E_0) + (1-\alpha)\frac{\phi\rho}{\zeta\kappa}\frac{1}{\frac{\tau_t}{C_t}\frac{\phi\rho}{\zeta}}\left(e^{-\kappa t}\Psi_t^D - \Omega_t^D\right) & \text{if } \tau \neq 0 \\ (1-e^{-\kappa t})E_0 + e^{-\kappa t}P_0 + \phi S_0\left(1+\frac{\rho e^{-\kappa t}-\kappa e^{-\rho t}}{\kappa-\rho}\right) & \text{if } \tau = 0 \end{cases} \tag{40}$$

with

$$\Psi_t^D = \left(1-e^{-\frac{S_0\zeta}{1-\alpha}\frac{\tau_t}{C_t}\frac{\phi\rho}{\zeta}}\right)\sum_{n=0}^{\infty} n!\frac{\Gamma\left(\frac{\kappa}{\rho}+1\right)}{\Gamma\left(\frac{\kappa}{\rho}+1+n\right)}\frac{1}{e^{n\frac{S_0\zeta}{1-\alpha}\frac{\tau_t}{C_t}\frac{\phi\rho}{\zeta}}},$$

$$\Omega_t^D = \frac{1}{1-e^{\rho t}\left(1-e^{\frac{S_0\zeta}{1-\alpha}\frac{\tau_t}{C_t}\frac{\phi\rho}{\zeta}}\right)^{-1}}\sum_{n=0}^{\infty} n!\frac{\Gamma\left(\frac{\kappa}{\rho}+1\right)}{\Gamma\left(\frac{\kappa}{\rho}+1+n\right)}\frac{1}{\left(1-e^{-\rho t}\left(1-e^{\frac{S_0\zeta}{1-\alpha}\frac{\tau_t}{C_t}\frac{\phi\rho}{\zeta}}\right)\right)^n}.$$

Finally, growth rates follow the decentralized version of (12). From this we get the following

[12] Proof for P_t follows as in section 3 in Appendix A

Proposition 5 *With flexible carbon taxes*

(i) an optimally set carbon tax is proportional to the consumption good and can restore the social optimum,

(ii) a zero tax rate results in the fastest equilibrium resource extraction

(iii) the optimal tax rate τ^o depends on diffusion speed κ via the function ζ

(iv) myopically set tax rates (when perceived κ is smaller than true κ or perceived χ is smaller than true χ) are too low compared to optimum tax rates.

Proof: see Appendix C

The optimal emissions tax rate $\tau^o = \frac{\zeta}{\phi\rho}C$ depends on κ through the parameter $\zeta = \zeta(\kappa) = \frac{\phi\chi\alpha}{\rho(\frac{\rho}{\kappa}+1)}$, the pollution intensity parameter, see Figure 3. For example, a slower diffusion (small κ) calls for a lower tax in a social optimum. The higher the tax is, the lower non-renewable resource extraction becomes. Resource use is stretched to the future and so does pollution accumulation. The economy then follows a higher transition path towards its steady state. Zero tax would result in fast extraction and use of the polluting non-renewable resources (follows from (38)) and in the fastest occurrence of maximum pollution. The economy will then follow a sub-optimal path towards the steady state and the growth rate will reach its lowest value faster (follows from (35), (38), (40)), see figure 4. It can be derived additionally that the inclusion of a clean input like in section 4 changes the results in the same way as before; resource use and the optimum emissions tax would be lower.

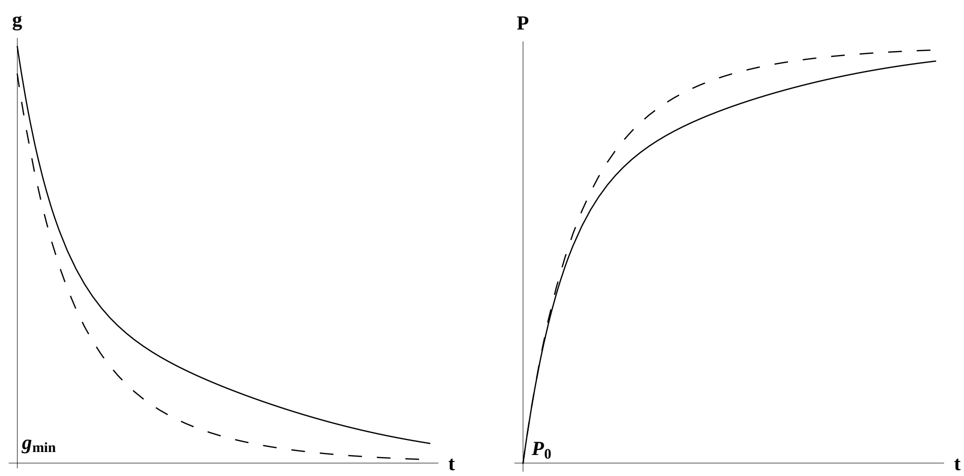

Figure 4: Growth rate of the economy and pollution with $\tau = \tau^o$ (solid) and $\tau = 0$ (dashed)

If policy makers poorly understand the true emission dissemination speed (κ) or other model parameters the chosen tax rate deviates from the optimum. There are mainly three reasons to suggest that actual policies have a strong tendency to set too low tax rates in this context. First, policy makers have a tendency to postpone unpopular measures because of (re-)election constraints. Hence, they like to believe that the true parameter value for emission dissemination is $\tilde{\kappa}$ with $\tilde{\kappa} < \kappa$ so that the actually set tax rate τ is smaller than τ^o. Because climate damages come with a delay, voters do not necessarily sanction such a behavior. Second, because of delayed climate costs, decision makers might erroneously think that damage intensity χ is $\tilde{\chi}$ (with $\chi > \tilde{\chi}$) because the damage they actually observe does not encompass the entire damage which is yet to occur. Then, the chosen tax rate will again be lower than in optimum because χ positively affects τ^o via ζ. Finally, the same occurs when policy makers apply a higher discount rate $\tilde{\rho} > \rho$ in their political decisions thereby weighting less future generations. Such misperceptions can easily emerge because invisible future damages are a difficult argument in the rather myopic political debates. The decisions on policies have to rely heavily on scientific forecasts which are not always readily accepted. In such a case, the resulting path in figure 4 would lie between the solid and the dashed line. We conclude that with a too low tax rate, at each point in time resource extraction and subsequent harmful pollution is higher and aggregate welfare lower than optimum if policy makers are myopic.

Figure 5 shows the effect of setting a too low tax rate to the emissions and harming pollution accumulation. Setting a too low tax rate as a result of the misperception about the relevant parameters κ or χ would lead to faster resource extraction and higher emissions and harmful pollution accumulation.

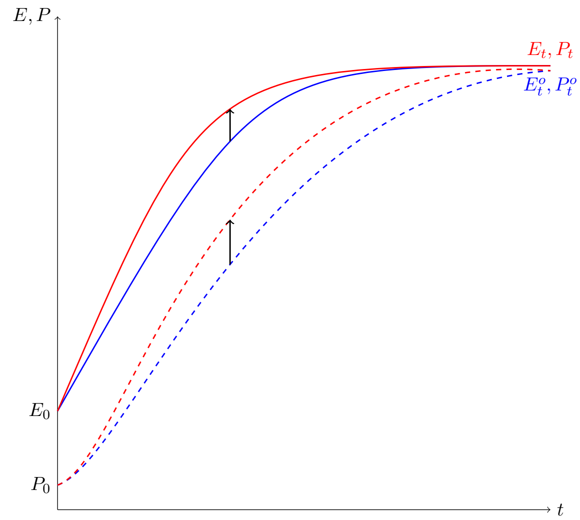

Figure 5: Effect of policy on accumulated emissions (E) and harmful pollution (P). Blue lines optimal taxation, red lines too low taxes.

5.3 Discussion

We have derived that a constant Pigouvian tax τ^o, given by (36), restores the first best allocation in an economy with pollution lags. A constant optimal tax per unit of consumption has also been found in other dynamic pollution models, see Grimaud and Rouge (2014), Golosov et al. (2014), Gerlagh and Liski (2012) for example. The important point about the constant tax rate/consumption ratio is that it provides the correct resource extraction incentives to the economy. To be more precise, given finite resource stocks, it is the growth rate of the tax and not its level that matters, see Dasgupta and Heal (1979) and Gaudet and Lasserre (2013). In Appendix C we characterize a generalization of the optimal tax rate in our model.

The optimal carbon tax that postpones extraction has to grow at a slower rate than the price of the non-renewable resource. Then, the unit price paid for the resource by consumers increases less rapidly than the price received by producers, which grows at the market's (consumption denominated) interest rate, giving them the incentive to postpone extraction. In fact this implies an equivalence between the constant per unit of emissions tax as in our case and a decreasing ad-valorem tax. To see that, note that from (31) the unit cost for the resource of the firm producing the consumption good is $p_{R,t} + \phi\tau_t = p_{R,t}\left(1 + \phi\frac{\tau_t}{p_{R,t}}\right)$. The price received by producers, $p_{R,t}$ grows at the rate $r_{C,t}$ while the tax rate grows at $r_{C,t} - \rho$, implying a decreasing ad-valorem tax rate.

The optimal tax at each point in time, that restores the first best allocation, is determined as the utility denominated discounted sum of all marginal damages from that point on in the economy. Had we assumed no lag in the climate system and a linear separable damage function in the utility function as in Grimaud and Rouge (2014), or an exponential damage function in a multiplicative way in the production of the consumer's good (1) as in Golosov et al. (2014)[13], the optimal tax in our model would read $\tau^o = \frac{\chi}{\rho}C$ (with a pollution decay parameter ψ, the optimal tax would change to $\tau^o = \frac{\chi}{\rho+\psi}C$, the result in Grimaud and Rouge (2014) with χ measuring the impact of pollution on the utility. We will not consider this case any longer since our results are robust with a minor adjustment to include the parameter θ). When no lags are accounted for and climate change is responsible for deteriorating the existing capital stock as in our case, the optimal tax can be shown to be $\tau^o = \frac{\alpha\chi}{\rho^2}C$. The parameter α in the numerator accounts for the marginal product of capital and the extra ρ in the denominator comes about because capital is a state variable and additional discounting at the rate ρ at each time period is needed.

In the same logic we can calculate the optimal tax when accounting for a time lag in the climate system. When pollution acts negatively in a linear and separable way in the utility function of the representative consumer, the optimal tax is defined as $\tau^o = -\frac{1}{U_C}\int_t^\infty \left(\int_{-\infty}^s \kappa e^{-(\kappa+\rho)(v-s)} D'(P_v)dv\right) e^{-\rho(s-t)}ds$. The term $\int_{-\infty}^s \kappa e^{-(\kappa+\rho)(v-s)} D'(P_v)dv$ accounts for the marginal damage of an additional unit of lagged harmful pollution due to an additional unit of accumulated greenhouse gas emissions. The term $e^{-\rho(s-t)}$ is responsible for total discounting to time t, as usual in economics. The first term, $\frac{1}{U_C} = C$, is responsible for making the expression utility denominated. The result of that expression, since $D'(P) = \chi$, is $\tau^o = \frac{\chi}{\rho\left(\frac{\rho}{\kappa}+1\right)}C$. Accounting for the fact that pollution is destroying the existing capital stock results to $\tau^o = \frac{\alpha\chi}{\rho^2\left(\frac{\rho}{\kappa}+1\right)}C$, as we showed before, i.e. equation (36).

6 Evidence and model application

The model relies on the assumption that natural disasters have a substantial impact on the economy destroying part of its capital stock. At the same time, economic growth exacerbates

[13] For an exposition on the economic interpretation of the optimal tax and on the equivalence of the two pollution damage modelling approaches, see Grimaud and Rouge (2014).

the impact of natural disasters as the economy accumulates capital so that each new event has a higher damaging potential in economic and in physical terms. It can be noted that, since 1900, reported economic damages related to weather phenomena and climate change such as floods, droughts, storms, extreme temperatures, and wildfires account for about 75% of all the natural disasters recorded (EM-DAT International Disasters Database - CRED). Moreover reported damages have increased severely since the late 1980s.

As derived in the model, optimum policy requires a correct perception of the crucial model parameters, notably the time lag for pollution dissemination. Figures 6 and 7 show the lag between CO2 emissions and the increase of global temperature for past development. Specifically, we see the evolution of CO2 emissions and the temperature anomaly compared to the reference time period 1961-1990. By using temperature as an indicator for climate change and taking linear trends, a lagged response of temperature to CO2 emissions of about 30 years can be assumed so far.

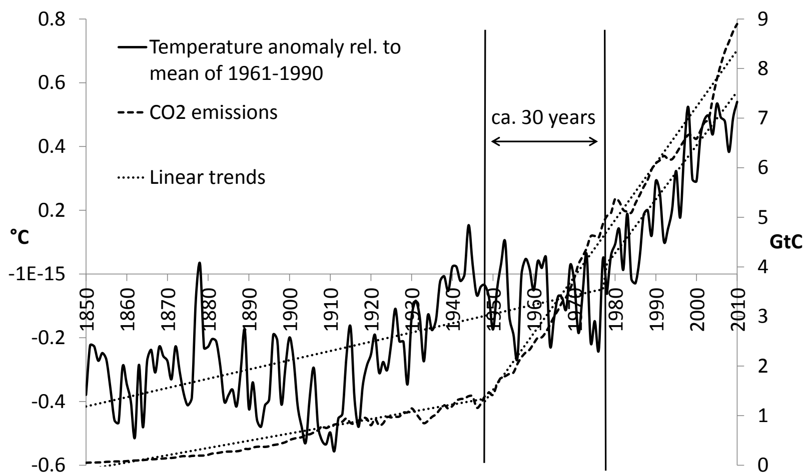

Figure 6: Empirical data of yearly CO2 emissions and Temperature anomaly relative to the mean of 1961-1990 for the period 1850-2010. Sources: CO2 emissions, Carbon Dioxide Information Analysis Center. Temperature anomaly, IPCC Data Distribution Center.

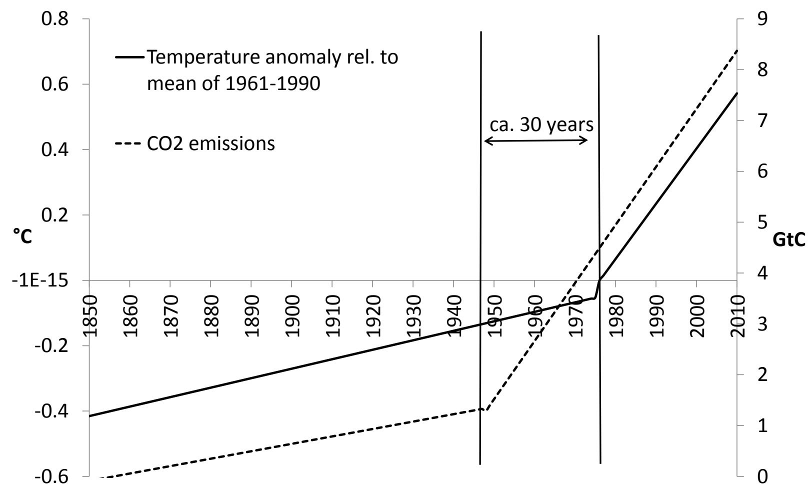

Figure 7: Identified linear trends of yearly CO2 emissions and Temperature anomaly relative to the mean of 1961-1990 for the period 1850-2010. Sources: CO2 emissions, Carbon Dioxide Information Analysis Center. Temperature anomaly, IPCC Data Distribution Center.

Looking into the future and the potentially large damages of climate change, e.g. following the Stern Review, one would expect a longer time lag of about 50 to 150 years, depending on the scenario followed (Stern (2007), Fig.6.5 a-d p.178). A certain degree of uncertainty remains in any case. A current example for the possible misperception of the carbon cycle is prominently given in the new IPCC fifth assessment report explaining "...due to natural variability, trends based on short records are very sensitive to the beginning and end dates and do not in general reflect long-term climate trends. As one example, the rate of warming over the past 15 years [...] is smaller than the rate calculated since 1951", see IPCC (2013 p.3.).

7 Conclusions

We use an endogenous growth model to study the effects of climate change which is induced by non-renewable resource use and follows a well-defined time pattern. The central feature of the paper is the inclusion of a lag between the stock of accumulated emissions and the stock of harmful pollution through the dissemination speed for which we provide closed-form solutions. The time lag between emissions and their effect on the economy, here on capital accumulation, has in general drawn little attention regardless its importance. The standard assumption in the literature of an instantaneous diffusion is the limiting case in our model.

We find that the emission dissemination speed has a crucial impact on growth and welfare in the social optimum. Specifically, the optimal per unit emission tax rate increases in dissemination speed so if emission taxes are not set by the social planner but by a regular political process the risk of setting tax rates too low is imminent when actors underestimate the true pollution dissemination speed. Then the economy follows a sub-optimal path towards its steady state. The availability of clean energy inputs enlarges the scope for development; then, economic growth depends on the share of the clean energy technology in the production of the consumption good.

Underestimation of climate change and pollution dissemination has different reasons. The usually observed myopia of decision makers and short-run targets like elections are one component. Moreover, climate sciences provide results and predictions which naturally include a certain degree of uncertainty because they concern the very long run. Finally, reactions and decisions might rely on cognitive experience. When environmental damages become visible they have the best conditions to trigger political action. Because this is not (yet) the case for climate change, the concerns of too little political action appear to be warranted.

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A Appendix - Social planner solution

The social planner maximizes (9) subject to (1), (3), (4), (8). The Hamiltonian of this program reads (we denote with $g_{z,t} \equiv \hat{z}_t \equiv \frac{\dot{z}_t}{z_t}$ the growth rate of z_t)

$$H = \ln(F(\epsilon_t K_t, R_t)) + \lambda_t [B(1-\epsilon)K_t - \chi P_t K_t] - \mu_t R_t + \nu_t \kappa [E_0 + \phi(S_0 - S_t) - P_t]$$

Assuming an interior solution, the first order conditions for ϵ and R are

$$\frac{\partial H}{\partial \epsilon_t} = \frac{\alpha}{\epsilon_t} - B\lambda_t K_t = 0 \tag{A.1}$$

$$\frac{\partial H}{\partial R_t} = \frac{1-\alpha}{R_t} - \mu_t = 0 \tag{A.2}$$

The first order conditions for the state variables are

$$\frac{\partial H}{\partial K_t} = \frac{\alpha}{K_t} + \lambda_t \widehat{K}_t = \rho\lambda_t - \dot{\lambda}_t \tag{A.3}$$

$$\frac{\partial H}{\partial S_t} = -\kappa\phi\nu_t = \rho\mu_t - \dot{\mu}_t \tag{A.4}$$

$$\frac{\partial H}{\partial P_t} = -\chi(\lambda_t K_t) - \kappa\nu_t = \rho\nu_t - \dot{\nu}_t \tag{A.5}$$

Furthermore there is one transversality condition for each state variable, S, K, P

$$\lim_{t\to\infty} (\lambda_t K_t) e^{-\rho t} = 0 \tag{A.6}$$

$$\lim_{t\to\infty} (\mu_t S_t) e^{-\rho t} = 0 \tag{A.7}$$

$$\lim_{t\to\infty} (\nu_t P_t) e^{-\rho t} = 0 \tag{A.8}$$

Equation (A.3) combined with (A.6) gives $\lambda_t K_t = \alpha/\rho$ so that from (A.1) $\epsilon = \frac{\rho}{B}$. Then from (A.5) and (A.8) we get that $\nu = -\frac{\chi\alpha}{\rho(\rho+\kappa)}$. From (A.4) and (A.7) with $\zeta = \frac{\phi\chi\alpha}{\rho(\frac{\rho}{\kappa}+1)}$ we get the function of the shadow price of the non-renewable resource stock $\mu_t = \frac{\zeta}{\rho} + e^{\rho t}\left(\mu_0 - \frac{\zeta}{\rho}\right)$. Using (A.2) with the condition that the whole resource stock has to be depleted eventually, $\int_0^\infty R_t dt = S_0$, we get that $\mu_0 = \frac{\zeta}{\rho}\left(\frac{1}{1-e^{-\frac{S_0\zeta}{1-\alpha}}}\right)$. Finally with the initial value of the shadow price of the non-renewable resource stock we can calculate resource extraction as in (13) of the main text. Equation (14) comes from solving (5).

For the solution of (7) with E_t given by (14) we make use of the Mathematica software. After some simplifications the result reads:

$$P_t = E_t + e^{-\kappa t}(P_0 - E_0) + (1-\alpha)\frac{\phi\rho}{\zeta\kappa}\left[e^{-\kappa t}\mathcal{F}\left(1, \frac{\kappa}{\rho}, \frac{\kappa}{\rho}+1, \frac{1}{1-e^{\frac{S_0\zeta}{1-\alpha}}}\right) - \mathcal{F}\left(1, \frac{\kappa}{\rho}, \frac{\kappa}{\rho}+1, \frac{e^{\rho t}}{1-e^{\frac{S_0\zeta}{1-\alpha}}}\right)\right]$$

$$\tag{A.9}$$

where the function $\mathcal{F}$ refers to the hypergeometric function

$$ {}_2F_1(a,b,c,z) = \sum_0^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!}, \qquad (q)_n = \begin{cases} 1 & \text{if } n = 0 \\ q(q+1)(q+2)...(q+n-1) & \text{if } n > 0 \end{cases} $$

as an approximate solution to a second-order linear ordinary differential equation. For $|z| < 1$ the series converges. In order to easier prove convergence and to have a more tractable solution we use the Pfaff transformation

$$\mathcal{F}(a,b,c,z) = (1-z)^{-a} \mathcal{F}\left(a, c-b, c, \frac{z}{z-1}\right) \tag{A.10}$$

which gives

$$P_t = E_t + e^{-\kappa t}(P_0 - E_0) + (1-\alpha)\frac{\phi\rho}{\zeta\kappa}\left[e^{-\kappa t}\Psi_t - \Omega_t\right] \tag{A.11}$$

with

$$\Psi_t = \left(1 - e^{-\frac{S_0\zeta}{1-\alpha}}\right) \mathcal{F}\left(1, 1, \frac{\kappa}{\rho} + 1, \frac{1}{e^{\frac{S_0\zeta}{1-\alpha}}}\right)$$

$$\Omega_t = \frac{1}{1 - e^{\rho t}\left(1 - e^{\frac{S_0\zeta}{1-\alpha}}\right)^{-1}} \mathcal{F}\left(1, 1, \frac{\kappa}{\rho} + 1, \frac{1}{1 - e^{-\rho t}\left(1 - e^{\frac{S_0\zeta}{1-\alpha}}\right)}\right)$$

By the definition of the hypergeometric function the series in both Ψ and Ω functions converge for $\zeta > 0$. After some calculations and noting that $\left(\frac{\kappa}{\rho} + 1\right)_n$ can be the product of the n terms of an arithmetic progression with $\alpha_n = \left(\frac{\kappa}{\rho} + n\right)$ and common difference 1, we arrive at the result of (15). The same procedure is followed for the calculation of the functional form of pollution in sections 4 and 5.

B Appendix - Clean Energy

The social planner maximizes (9) subject to (19), (20), (21), (4), (8). The Hamiltonian of this program reads (we denote with $g_{z,t} \equiv \hat{z}_t \equiv \frac{\dot{z}_t}{z_t}$ the growth rate of z_t)

$$H = \ln(F(\epsilon_{C,t}K_t, R_t, M_t)) + \lambda_t K_t\left[B(1-\epsilon_{C,t}-\epsilon_{M,t}) - \chi P_t\right] - \mu_t R_t + \nu_t \kappa\left[E_0 + \phi(S_0 - S_t) - P_t\right] + \xi_t\left[\tilde{B}(\epsilon_{M,t} - \theta)K_t - \chi P_t M_t\right]$$

Assuming an interior solution, the first order conditions for ϵ_C, ϵ_M and R are

$$\frac{\partial H}{\partial \epsilon_{C,t}} = \frac{\alpha}{\epsilon_{C,t}} - B\lambda_t K_t = 0 \tag{B.1}$$

$$\frac{\partial H}{\partial \epsilon_{M,t}} = B(\lambda_t K_t) - \tilde{B}(\xi_t K_t) = 0 \tag{B.2}$$

$$\frac{\partial H}{\partial R_t} = \frac{\beta - \gamma}{R_t} - \mu_t = 0 \tag{B.3}$$

The first order conditions for the state variables are

$$\frac{\partial H}{\partial K_t} = \frac{\alpha}{K_t} + \lambda_t \widehat{K}_t + \xi_t \tilde{B}(\epsilon_{M,t} - \theta) = \rho\lambda_t - \dot{\lambda}_t \tag{B.4}$$

$$\frac{\partial H}{\partial M_t} = \frac{\gamma}{M_t} - \xi_t \chi P_t = \rho\xi_t - \dot{\xi}_t \tag{B.5}$$

$$\frac{\partial H}{\partial S_t} = -\kappa\phi\nu_t = \rho\mu_t - \dot{\mu}_t \tag{B.6}$$

$$\frac{\partial H}{\partial P_t} = -\chi(\lambda_t K_t) - \chi(\xi_t M_t) - \kappa\nu_t = \rho\nu_t - \dot{\nu}_t. \tag{B.7}$$

Furthermore there is one transversality condition for each state variable, S, K, M, P

$$\lim_{t\to\infty} (\lambda_t K_t)e^{-\rho t} = 0 \tag{B.8}$$

$$\lim_{t\to\infty} (\xi_t M_t)e^{-\rho t} = 0 \tag{B.9}$$

$$\lim_{t\to\infty} (\mu_t S_t)e^{-\rho t} = 0 \tag{B.10}$$

$$\lim_{t\to\infty} (\nu_t P_t)e^{-\rho t} = 0. \tag{B.11}$$

From (B.1) and (B.2) is evident that

$$\widehat{\lambda_t K_t} = \widehat{\xi_t K_t} = -\widehat{\epsilon}_{C,t} \qquad \rightarrow \qquad \xi_t K_t = \frac{\alpha}{\tilde{B}\epsilon_{C,t}}, \qquad \lambda_t K_t = \frac{\alpha}{B\epsilon_{C,t}} \tag{B.12}$$

By combining (B.4) with (B.1) and (B.2), (B.5) with (21) and (B.1), (21) with (19) and (21) with (B.5) we get respectively

$$-\widehat{\epsilon}_{C,t} = \rho - B(\epsilon_{C,t} + \epsilon_{M,t}) + \theta B \tag{B.13}$$

$$-\widehat{\epsilon}_{C,t} = \rho - \frac{\gamma\tilde{B}}{\alpha}\epsilon_{C,t}\frac{K_t}{M_t} + B(1 - \epsilon_{C,t} - \epsilon_{M,t}) \tag{B.14}$$

$$\widehat{\left(\frac{K_t}{M_t}\right)} = B(1 - \epsilon_{C,t} - \epsilon_{M,t}) - \tilde{B}(\epsilon_{M,t} - \theta)\frac{K_t}{M_t} \tag{B.15}$$

$$\widehat{(\xi_t K_t)} = \rho - \frac{\gamma}{\xi_t K_t}\left(\frac{K_t}{M_t}\right) + B(1 - \epsilon_{C,t} - \epsilon_{M,t}). \tag{B.16}$$

Equating (B.13) with (B.14) gives

$$\frac{\gamma\tilde{B}}{\alpha B}\left(\frac{K_t}{M_t}\right) = \frac{1-\theta}{\epsilon_{C,t}} \tag{B.17}$$

and using (B.15) we get

$$\frac{\epsilon_{M,t}-\theta}{\epsilon_{C,t}} = \frac{\gamma}{\alpha}\left(1-\frac{\rho}{B(1-\theta)}\right), \tag{B.18}$$

which with (B.13) results in $-\widehat{\epsilon}_{C,t} = \rho - \epsilon_{C,t}B\left(1+\frac{\gamma}{\alpha}\left(1-\frac{\rho}{B(1-\theta)}\right)\right)$. The solution to that differential equation taking into account the transversality condition (B.8) results in ϵ_C of (22) in the main text. Parameter ϵ_M follows from (B.18). Since capital shares jump immediately to their steady state, from (B.17), $\frac{K_t}{M_t}$ is constant. Thus K and M grow together leading to (23). (B.7) with (B.12), (22) and the transversality condition (B.11) gives $\nu = -\frac{\chi(\alpha+\gamma)}{\rho(\rho+\kappa)}$ and then following the same procedure as in Appendix A we get (24), (25) and (26) with $\tilde{\zeta} = \frac{\phi\chi(\alpha+\gamma)}{\rho(\frac{\rho}{\kappa}+1)}$.

C Appendix - Optimal tax rate

In this appendix we provide a generalization of the optimal tax rate of (36). Using (33) we get the time evolution of consumption

$$C_t = C_0 e^{\int_0^t (r_{C,s}-\rho)ds}, \tag{C.1}$$

and from (34) the time evolution of the price received by the producers of the non-renewable resource

$$p_{R,t} = p_{R,0}e^{\int_0^t r_{C,s}ds}. \tag{C.2}$$

Then $\frac{p_{R,t}}{C_t} = \frac{p_{R,0}}{C_0}e^{\rho t}$. Using that and (31) we get the resource extraction path

$$R_t = \frac{1-\alpha}{\phi\frac{\tau_t}{C_t}+\frac{p_{R,0}}{C_0}e^{\rho t}}. \tag{C.3}$$

At this point we have to restrict ourselves regarding the functional form of the tax rate. It is a known result from the theory of non-renewable resource taxation that any term in the optimal tax that grows with the (consumption denominated) interest rate has no effect on the extraction behavior of the economy. We can then set $\tau_t = \tilde{\tau}_t + Ne^{\int_0^t r_{C,s}ds}$ so that using (C.1), the resource extraction (C.3) now reads $R_t = \frac{1-\alpha}{\phi\frac{\tilde{\tau}_t}{C_t}+\frac{p_{R,0}+\phi N}{C_0}e^{\rho t}}$. In order to be able to solve the model analytically we impose beforehand a constant $\frac{\tilde{\tau}_t}{C_t}$ ratio and calculate the term $\frac{p_{R,0}+\phi N}{C_0}$

using the constraint $\int_0^\infty R_t dt = S_0$. By comparing the solution to the resource extraction of the social planner (13) we can verify that the expression

$$\tau_t^o = \frac{\zeta}{\phi\rho} C_t + N e^{\int_0^t r_{C,s} ds} \tag{C.4}$$

restores the social optimum (with the superscript ^{o} we define the optimum level). We have thus proven that there is a continuum of taxes having the same dynamics which give the same extraction incentives to the economy. In order to fully characterize the set of optimal taxes, giving the first best allocation, we have to set $N > -p_{R,0}/\phi$. Choosing then the optimal tax rate with $N = 0$ from the set of admissible optimal taxes leads to (36). Log-differentiating (37) gives (38). Accordingly one can calculate the emissions and harmful pollution accumulation following the same procedure as in the previous appendices.