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Abstract

In this paper, we propose to analyse the choice of risk management activity made by a non-industrial private forest owner who derives utility from consumption and from the sentimental value of the forest that bears a risk of disaster. We consider a bivariate utility function depending on consumption and sentimental value of forest. In this context, we analyse insurance and/or self-insurance decisions. We show that, under fair premium, full insurance is optimal only if the cross derivative of the utility function equals zero. Under-insurance and over-insurance may also be optimal depending on the sign of this cross derivative. We also show that, under a positive loading factor, optimal partial insurance is validated only if the cross derivative is positive; otherwise full insurance may be optimal even with a loading insurance. We also observe that risk aversion increases the level of insurance demand and self-insurance activity, extending this standard result obtained with an univariate utility function to a bivariate utility function.

Moreover, when the forest owner can simultaneously insure and invest in self-insurance activity, full insurance is never optimal if the cross derivative is positive. Finally, we prove that insurance and self-insurance may be substitutes, and if preferences are separable and exhibit decreasing absolute risk aversion, then insurance and self-insurance are always considered as substitutes.

Key words: Forest management, insurance, self-insurance, bivariate utility, risk.

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1 Introduction

Many works on forest economics have gotten interested on the behaviors of non-industrial private forest (NIPF) owners, mainly focusing on the decision of harvesting based from the view point of timber production, and market. The primary focus on harvesting according to the only utility from consumption with the revenue due to forest production may constitute an incomplete description of the forest owners' behaviors because some forest owners may be less interested in purely market-based activities and more interested in participating in non-timber activities on their land, such as, for example, hiking, landscape, and wildlife observation. Thus, some NIPF owners may have positive utility due to amenity functions of forests, produced jointly with timber and vanishing with the standing stock, along with the revenue from timber sales. It has been recently validated that NIPF owners confer some private values on the amenity services of forest stock, even if there are no financial incentives linked to these functions (Birch [8], Butler and Leatherberry [15], Zhang et al. [47]). Therefore, non-timber activities could be important in determining whether a forest owner will harvest or not, and also will manage risk faced by her/his forest. Indeed, NIPF owners face natural risks directly affecting the timber production of forests and, the amenity services to be obtained from standing stock (Schelhaas et al. [43]). Amenities therefore reinforce the interest of risk management measures. In such a context, the risk preferences of the forest owners are proved to be determinant (Lönnstedt and Svensson [36]; Andersson and Gong [5]; Brunette et al. [9]). Risk aversion is always a feature of optimal forest management. In particular, risk preferences strongly affect some decisions of forest management such as the rotation date or the planting, and other decisions linked to forest activities such as forest investment. Results indicate that risk aversion shorten the rotation lenght (Clarke and Reed [18]; Gong and Löfgren [27]; Alvarez and Koskela [2]). Moreover, the decisions of forest investments (Taylor and Forston [46]; Kangas [31]), and of replanting or not after a clear cutting also depend on the forest owner's risk preferences (Lien et al. [35]).

Forest risk management strategies can be divided into two main categories: thoses that involve risk sharing, and those that do not. The main risk sharing strategy that may be considered is insurance. Indeed, in several countries, insurance contracts against fire and/or storm are proposed to NIPF owners (Matsushita et al. [38]; Holecy and Hanewinkel [29]; Brunette and Couture [11]; Manley and Watt [37]; Barreal et al. [7]). Risk management strategies that do not involve risk sharing are common in forestry and are numerous. The simplest, and most basic, actions that can be implemented to face risks are those reducing the damages that a given hazard would cause. Such actions refer to self-insurance activities by economists (Ehrlich and Becker [23]). These activities are designed to mitigate the risks that impinge upon optimal forest management. They are very typical risk management strategies such as for example, internal fire tracks, reduction of rotation length, implementation of fencing and other natural barriers around the edges of a forest (Manley and Watt [37]), control burning (Amacher et al. [4]) among others. In a more general context, insurance and self-insurance are proved to be substitutes (Ehrlich and Becker [23]). More, amenities may have an impact on these risk management strategies. For instance, Brunette and Couture [12] found that the larger the marginal utility of amenities, the larger the physical and financial savings. Consequently, several questions emerge: How the introduction of amenities in the utility function, in addition to utility from consumption, affects the optimal decision of risk management activities ? Are self-insurance and insurance substitutes when amenities are taken into account ? What is the role of risk preferences on risk management decisions with a bivariate utility function ?

The effects of risk and amenity preferences on timber supply were previously analysed in the rotation model (Englin, Boxall and Hauer [25]) and in the two-period model (Koskela and Ollikainen [33]; Amacher, Ollikainen and Koskela [4]; Brunette and Couture [12]). These studies have shown that the harvesting rule is defined by a tradeoff between harvesting revenue and amenity valuation. However, these papers were not interested in NIPF owners' insurance behavior, and as a consequence they do not deal with a potential substitutability/complementarity between insurance and self-insurance. More importantly, a main assumption in these papers, but also more generally in forest economics (see Koskela and Ollikainen [32]; Amacher and Brazee [3]), is that utility is separable in timber and non-timber components. The use of a bivariate utility function is thus, to our knowledge, never considered in this literature. We take these two points in turn.

First, insurance decisions gathers a large literature since the early works of Arrow [6] and Mossin [40]. The main conclusion of these papers is that a risk-averse agent who maximizes her/his expected utility of wealth chooses full insurance if the insurance premium displays no loading factor (fair premium), and partial insurance otherwise (Arrow [6], Mossin [40]). In addition, Ehrlich and Becker [23] proved that insurance and self-insurance are substitutes. These results are validated in a forest framework with an univariate utility function (Brunette and Couture [10], [11]; Brunette, Couture and Garcia [13]).

Second, the multivariate utility function appeared for the first time in Eisner and Strotz [24] with pecuniary assets as arguments. The idea to consider a non-pecuniary asset as an argument in a bivariate utility function appeared at the end of 70's with the literature on 'irremplacable commodities' such as family snapshots, the family pet, good health, the life of a beloved spouse or child, etc., i.e. commodity that holds sentimental value for its owner in addition to market value (Cook and Graham [19]). The seminal paper of Cook and Graham [19] was the beginning of a long series of articles on this topic, see for example Mavromaras [39], Shioshansi [45], Schlesinger [44] or more recently, Huang and Tzeng [30]. This bivariate utility function is then applied to various economic fields such as health economics where it is widely used (Rey [42]; Eeckhoudt, Rey and Schlesinger [22]; Etner and Spaeter [26]). This literature highlights that the cross derivative of the bivariate utility function plays a significant role. However, to our knowledge, such a bivariate utility function had never been applied to forest economics to analyse utility coming from timber and non-timber benefits.

Consequently, our objective is to analyse the NIPF owner's insurance and self-insurance decisions considering a bivariate utility function. We propose a static analytical model of decision under risk taking into account risk preferences. First, we analyse separately insurance and self-insurance decision in order to test the validity of the existing results in our particular framework and then, we analyse optimal insurance with self-insurance activities. Our results are very sensitive to the cross derivative between consumption and sentimental value of the forest (U_{12}). We show that the Mossin's result of optimal full insurance with fair premium is true only if this cross derivative equals zero. In addition, under-insurance and over-insurance may also be optimal depending on the sign of this cross derivative. We also show that the Mossin's result of optimal partial insurance with a positive loading factor is validated only if the cross derivative is positive; meaning that full insurance may be optimal even with a loading insurance. We also observe that risk aversion increases the level of insurance demand and self-insurance activity, extending the standard result to a bivariate utility function. Moreover, when the forest owner can simultaneously insure and invest in self-insurance activity, full insurance is never optimal if the cross derivative is positive. Finally, we prove that insurance and self-insurance may be substitutes, and if preferences are separable and exhibit decreasing absolute risk aversion (DARA), then insurance and self-insurance are always considered as substitutes.

The rest of the paper is organised as follows. Section 2 presents the theoretical model. Section 3 concludes.

2 The model

We consider a static model in which a NIPF owner derives utility from consumption and from the sentimental value of the forest, i.e. non-pecuniary sentimental value linked to the existence of the forest stand. The forest owner's preferences are described by a bivariate utility function $U(C, S)$ with C being the consumption and S the value of the forest. U is increasing and concave in each term[1] $U_1 > 0$, $U_{11} < 0$, $U_2 > 0$, $U_{22} < 0$. We do not impose restriction on U_{12}, the cross derivative of U, so that the occurrence of a loss in forest can increase, decrease or can leave the forest owner's marginal utility of consumption unchanged. Eeckhoudt, Rey and Schlesinger [22] refer to a correlation averse individual if $U_{12} \leq 0$.

The consumption of the forest owner depends directly on the revenue of harvesting. We assume that the forest owner possesses an even-aged forest that provides an optimal revenue R that corresponds to the commercial value of a timber stand at the optimal cut period. This revenue is subject to natural risks and then to a possible loss. This risk appears with a probability p, and the loss is a proportion of the revenue: $L = xR$, with x in $[0, 1]$. The risk affects also the sentimental value of the forest. Indeed, if the riks occurs, both timber and non-timber benefits will be impacted. Then the loss for this component is h.

2.1 Optimal insurance

The private forest owner can purchase a co-insurance policy. This insurance contract consists of an indemnity function where the private forest owner receives payment αL in the event of a loss L, as well as a premium P, which must be paid no matter what. The forest owner chooses α between 0 and 1. The premium for the given indemnity function takes the form $P(R) = (1 + \lambda)\,\alpha\mu(R)$ with $\mu(R) = E[L] = pxR$ where $\lambda \geq 0$ is the loading factor. We assume that there is no moral hazard problem so that the forest owner will not be more careless in forest management as a result of purchasing insurance. Furthermore, we assume that the insurer has the same information about

[1]Index i denotes a partial derivative of the utility function with respect to argument i ($i = 1$ *or* 2).

risk as the forest owner, so that there will be no adverse selection issue.

Concerning insurance, the private forest owner has to choose α to maximize her/his expected utility:

$$Max_{\{\alpha\}}\ V(\alpha) = pU(R - xR + \alpha xR - (1+\lambda)\alpha pxR, S - h) + (1-p)U(R - (1+\lambda)\alpha pxR, S) \quad (1)$$

In case of a fair insurance premium ($\lambda = 0$), then the problem is the following one:

$$Max_{\{\alpha\}}\ U(\alpha) = pU(R - xR + \alpha xR - \alpha pxR, S - h) + (1-p)U(R - \alpha pxR, S)$$

Proposition 1 *In case of a fair insurance premium, the optimal insurance level depends on the sign of* U_{12}.

- *If* $U_{12} < 0$, *then the forest owner prefers over-insurance.*
- *If* $U_{12} = 0$, *then the forest owner prefers full insurance.*
- *If* $U_{12} > 0$, *then the forest owner prefers under-insurance.*

Proof.

The first-order condition is:

$$\frac{dU(\alpha)}{d\alpha} = pU_1(R - xR + \alpha xR - \alpha pxR, S - h)(xR(1-p)) - (1-p)U_1(R - \alpha pxR, S)(pxR) = 0$$

This equation is equivalent to:

$$U_1(R - xR + \alpha xR - \alpha pxR, S - h) = U_1(R - \alpha pxR, S)$$

To determinate the value of α^* we evaluate the first order condition for full insurance ($\alpha = 1$). We obtain:

$$\frac{dU(\alpha)}{d\alpha}|_{\alpha=1} = p(1-p)xR \cdot [U_1(R - pxR, S - h) - U_1(R - pxR, S)]$$

As $p(1-p)xR > 0$, the sign of this expression depends on the sign of U_{12}.
If $U_{12} < 0$, then $U_1(R - pxR, S - h) > U_1(R - pxR, S)$ and $\frac{dU(\alpha)}{d\alpha}|_{\alpha=1} > 0$. We deduce that $\alpha^* > 1$.
If $U_{12} = 0$, then $U_1(R - pxR, S - h) = U_1(R - pxR, S)$ and $\frac{dU(\alpha)}{d\alpha}|_{\alpha=1} = 0$. We deduce that $\alpha^* = 1$.

If $U_{12} > 0$, then $U_1(R - pxR, S - h) < U_1(R - pxR, S)$ and $\frac{dU(\alpha)}{d\alpha}|_{\alpha=1} < 0$. We deduce that $\alpha^* < 1$.
■

With a fair insurance premium, full insurance is optimal only if the cross derivative of the utility function equals zero. This means that with a bivariate utility function depending on consumption and sentimental value of the forest, the famous Mossin's result[2] of optimal full insurance with fair premium is true only if $U_{12} = 0$, i.e. when the marginal utility of consumption is a constant function of the sentimental value of the forest. In addition, under-insurance and over-insurance may also be optimal decisions, depending on the sign of the cross derivative of the utility function U_{12}.

In case of a loading insurance premium, the problem of the forest owner is defined by equation (1).

The optimal level of co-insurance, α^*, satisfies:

$$\begin{aligned} \frac{dV(\alpha)}{d\alpha} &= pU_1(R - xR + \alpha xR - (1+\lambda)\alpha pxR, S - h)(xR - (1+\lambda)pxR)) \\ &- (1-p)U_1(R - (1+\lambda)\alpha pxR, S)((1+\lambda)pxR) = 0 \end{aligned} \tag{2}$$

This equation is equivalent to:

$$pxR(1-(1+\lambda)p)U_1(R-xR+\alpha xR-(1+\lambda)\alpha pxR, S-h) = pxR((1+\lambda)-(1+\lambda)p)U_1(R-(1+\lambda)\alpha pxR, S)$$

Proposition 2 *In case of a loading insurance premium, if $U_{12} \leq 0$ and $\lambda \leq \lambda_0$ with $\lambda_0 > 0$, full insurance is optimal. If $U_{12} > 0$, full insurance is never optimal.*

Proof. We evaluate the first order condition for full insurance ($\alpha = 1$). We obtain:

$$\frac{dV(\alpha)}{d\alpha}|_{\alpha=1} = pxR(1-(1+\lambda)p)U_1(R-(1+\lambda)pxR, S-h) - pxR((1+\lambda)-(1+\lambda)p)U_1(R-(1+\lambda)pxR, S)$$

If $U_{12} > 0$, then, $U_1(R-(1+\lambda)pxR, S-h) < U_1(R-(1+\lambda)pxR, S)$, and, as $pxR(1-(1+\lambda)p) < pxR((1+\lambda)-(1+\lambda)p)$, then $\frac{dV(\alpha)}{d\alpha}|_{\alpha=1}$ is always negative. Full insurance is never optimal.

If $U_{12} < 0$, then, $U_1(R - (1+\lambda)pxR, S - h) > U_1(R - (1+\lambda)pxR, S)$. Therefore, there exists

[2] Mossin [40], in a model of univariate utility function, proved that when the insurance premium is fair, then the optimal decision of a risk averse individual is to choose full insurance. A contrario, when the insurance premium is unfair, then the optimal decision is to choose partial insurance. These results are called the Mossin's theorem.

a λ_0 such as $\frac{dV(\alpha)}{d\alpha}|_{\alpha=1} = 0$. λ_0 is defined such as: $pxR(1-(1+\lambda)p)U_1(R-(1+\lambda)pxR, S-h) = pxR((1+\lambda)-(1+\lambda)p)U_1(R-(1+\lambda)pxR, S)$. Then for all $\lambda \leq \lambda_0$, $\frac{dV(\alpha)}{d\alpha}|_{\alpha=1} \geq 0$. Full insurance is always optimal.

■

Consequently, the Mossin's result of optimal partial insurance with a positive loading factor is validated in our model, only if $U_{12} > 0$, i.e. when the marginal utility of consumption is an increasing function of the sentimental value of the forest. Moreover, we show that considering the sentimental value of the forest as an additional argument in the utility function allows full insurance to be optimal even with a positive loading factor. Indeed, there exists a strictly positive scalar such that for any loading factor beyond this level, full insurance remains optimal.

Optimal insurance depends on the level of risk aversion that measures the resistance to fluctuations across states of nature. Therefore it is important to investigate the impact of increased risk aversion on optimal insurance decision. Intuition suggests that increased risk aversion induces the forest owner to take a lower level of risky activity and tends to increase optimal insurance decision. It is then interesting to study this impact.

Proposition 3 *Risk aversion increases the level of insurance.*

Proof. Let us consider a change of utility function from U to U^2. Suppose that there is an increasing and concave function g such that $U^2 = g(U)$.

The optimal level of insurance, α_2^*, for the forest owner with the U^2 utility function satisfies:

$$pg'(U(B,S-h))U_1(B,S-h)(xR-(1+\lambda)pxR)-(1-p)g'(U(A,S))U_1(A,S)((1+\lambda)pxR)=0 \quad (3)$$

with $B = R - xR + \alpha xR - (1+\lambda)\alpha pxR$, and $A = R - (1+\lambda)\alpha pxR$.

Since $g'' < 0$, we have $g'(U(B,S-h)) > g'(U(A,S))$. Using equation (2), it therefore follows that equation (3) will be positive when evaluated at α^*. Consequently, the optimal level of insurance under U^2 exceeds the optimal level of insurance under U, $\alpha_2^* > \alpha^*$.

■

Pratt [41] and Arrow [6], using an univariate utility function, proved that risk aversion increases the level of insurance. Our proposition extends this result to a bivariate utility function.

2.2 Optimal self-insurance activity

Here we analyse the possibility of forest management activity without insurance. q denotes self-insurance activity, and c is the unit cost of self-insurance. Self-insurance activity affects the loss such as $x(q)$ with $x_q < 0$ and $x_{qq} > 0$.

Concerning the self-insurance activity, the private forest owner has to choose q to maximize her/his expected utility:

$$Max_{\{q\}}\ \ W(q) = pU(R - x(q)R - cq, S - h) + (1 - p)U(R - cq, S) \tag{4}$$

The optimal level of self-insurance activity, q^*, satisfies:

$$\frac{dW(q)}{dq} = -pU_1(R - x(q^*)R - cq^*, S - h)(x_qR + c) - (1 - p)cU_1(R - cq^*, S) = 0 \tag{5}$$

At the optimal self-insurance level, we must have $x_qR + c < 0$, the potential marginal benefit must be at least as high as the cost of the increase in q.

Optimal self-insurance level depends on the level of risk aversion, so that it appears essential to analyse the effect of increased risk aversion on optimal self-insurance decision. As for insurance modelling, the intuition suggests that increased risk aversion leads the forest owner to take a lower level of risky activity and tends to increase optimal self-insurance decision.

Proposition 4 *Risk aversion increases the level of self-insurance activity.*

Proof. Let us consider a change of utility function from V to U. Suppose that there is an increasing and concave function g such that $V = g(U)$.

The optimal level of self-insurance activity, $\hat{q}$, for the forest owner with the V utility function satisfies:

$$\frac{dV(q)}{dq} = -pV_1(R - x(\hat{q})R - c\hat{q}, S - h)(x_qR + c) - (1 - p)cV_1(R - c\hat{q}, S) = 0 \tag{6}$$

Since $g'' < 0$, we have $g'(U(R - x(q)R - cq, S - h)) > g'(U(R - cq, S))$. Using equation (5), it therefore follows that equation (6) will be positive when evaluated at q^*. Consequently, the optimal level of self-insurance under V exceeds the optimal level of self-insurance under U, $\hat{q} > q^*$.

■

Dionne and Eeckhoudt [20] and Bryis and Schlesinger [14], using an univariate utility function, proved that risk aversion increases the level of self-insurance. Our proposition extends this result to a bivariate utility function.

2.3 Optimal insurance with self-insurance activity

In this section, we analyse the demand for insurance with the possibility of simultaneously investing in self-insurance. The forest owner has to choose the level of co-insurance α and the level of self-insurance q which maximize her/his utility. Self-insurance is observable by the insurer and is taken into account when the premium is computed, so that the premium and the indemnity are written as follows: $P = (1+\lambda)\alpha p x(q) R$ and $I = \alpha x(q) R$.

The forest owner computes now the expected utility:

$$V(\alpha, q) = pU(R - x(q)R + \alpha x(q)R - (1+\lambda)\alpha p x(q)R - cq, S - h) + (1-p)U(R - (1+\lambda)\alpha p x(q)R - cq, S) \tag{7}$$

The first-order conditions are:

$$H = \frac{\partial V(\alpha, q)}{\partial \alpha} = pU_1(B, S-h)(x(q)R - (1+\lambda)px(q)R)) - (1-p)U_1(A, S)((1+\lambda)px(q)R) = 0 \tag{8}$$

and

$$K = \frac{\partial V(\alpha, q)}{\partial q} = pU_1(B, S-h)(-x_q R + \alpha x_q R - (1+\lambda)\alpha p x_q R - c) - (1-p)U_1(A, S)((1+\lambda)\alpha p x_q R + c) = 0 \tag{9}$$

with $B = R - x(q)R + \alpha x(q)R - (1+\lambda)\alpha p x(q)R - cq$, and $A = R - (1+\lambda)\alpha p x(q)R - cq$.

We evaluate the first-order condition (8) for full insurance ($\alpha = 1$) and we obtain:

$$\frac{dV(\alpha, q)}{d\alpha}|_{\alpha=1} = px(q)R(1 - (1+\lambda)p)U_1(B|_{\alpha=1}, S-h) - px(q)R((1+\lambda) - (1+\lambda)p)U_1(A|_{\alpha=1}, S)$$

with $B|_{\alpha=1} = A|_{\alpha=1} = R - (1+\lambda)px(q)R - cq$.

Using the same principle of the proof of Proposition 2, we obtain:

Proposition 5 *When the forest owner can simultaneously buy co-insurance and invest in self-insurance activity, full insurance is optimal if* $U_{12} \leq 0$ *and* $\lambda \leq \lambda_1$ *with* $\lambda_1 > 0$*. If* $U_{12} > 0$*, full insurance is never optimal.*

Consequently, when the forest owner can simultaneously buy co-insurance and invest in self-insurance activity, full insurance is optimal only if the forest owner is correlation averse (Eeckhoudt, Rey and Schlesinger [22]). In such a case, there exists a strictly positive scalar such that for any loading factor beyond this level, full insurance remains optimal.

We now analyse the relation between insurance and self-insurance.

Proposition 6 *The forest owner may consider insurance and self-insurance as substitutes.*

Proof.

We analyse the impact of a variation of c on optimal insurance α^*. The differenciation of condition (8) gives:

$$\frac{d\alpha^*}{dc} = -\frac{\partial H/\partial c}{\partial H/\partial \alpha}$$

so that, the sign of $\frac{d\alpha^*}{dc}$ depends on $\frac{\partial H}{\partial c}$.

$$\begin{aligned}\frac{\partial H}{\partial c} &= -pU_{11}(B, S-h)(x(q)R - (1+\lambda)px(q)R)q + (1-p)U_{11}(A, S)((1+\lambda)px(q)R)q \\ &= (1+\lambda)px(q)Rq[(1-p)U_{11}(A, S) + pU_{11}(B, S-h)] - pU_{11}(B, S-h)x(q)Rq\end{aligned}$$

with $B = R - x(q)R + \alpha x(q)R - (1+\lambda)\alpha px(q)R - cq$, and $A = R - (1+\lambda)\alpha px(q)R - cq$. We know that $(1+\lambda)px(q)Rq > 0$, $U_{11} < 0$ so that the sign of this last expression is ambiguous.

Another solution consists in studying the impact of a variation of λ on optimal self-insurance q^*. The differenciation of condition (9) gives:

$$\frac{dq^*}{d\lambda} = -\frac{\partial K/\partial \lambda}{\partial K/\partial q}$$

so that, the sign of $\frac{dq^*}{d\lambda}$ depends on $\frac{\partial K}{\partial \lambda}$.

$$\frac{\partial K}{\partial \lambda} = -pU_{11}(B, S-h)(-x_q R + \alpha x_q R - (1+\lambda)\alpha p x_q R - c)(\alpha p x(q) R)$$
$$+(1-p)U_{11}(A, S)((1+\lambda)\alpha p x_q R + c)(\alpha p x(q) R)$$
$$= \alpha p x(q) R[(1-p)U_{11}(A, S)((1+\lambda)\alpha p x_q R + c) - pU_{11}(B, S-h)(-x_q R + \alpha x_q R - (1+\lambda)\alpha p x_q R - c)]$$

with $B = R - x(q)R + \alpha x(q)R - (1+\lambda)\alpha p x(q)R - cq$, and $A = R - (1+\lambda)\alpha p x(q)R - cq$.

We know that $\alpha p x(q) R > 0$, $U_{11} < 0$ so that the sign of this last expression is ambiguous.

■

Finally, we obtain that, if the preferences are represented by a bivariate utility function, then insurance and self-insurance may be substitutes. Indeed, the impact of an increase in the loading factor is ambiguous on optimal self-insurance decision. In the same manner, the effect of an increase in the cost of self-insurance is ambiguous on the optimal insurance level, so that the two risk management strategies may be considered as substitutes by the forest owner. This conclusion differs from the existing result obtained with an univariate utility function indicating that insurance and self-insurance are substitutes (Ehrlich and Becker [23]). When we introduce the utility of sentimental value, then this conclusion is not always validated.

To simplify the analysis, we assume, in line with much of the related literature (Drèze and Schokkaert [21]), that preferences are separable between both utility arguments, i.e. $U(C, S) = f(C) + g(S)$. The function $f(C)$ captures the effect of consumption on utility whereas the function $g(S)$ captures the effect of forest sentimental value on utility. We assume f and g to be continuously differentiable and strictly concave, i.e. $f' > 0$, $f'' < 0$, $g' > 0$, $g'' < 0$. In such a context, we obtain the following proposition:

Proposition 7 *If preferences are separable and exhibit DARA, then insurance and self-insurance are always considered as substitutes.*

Proof.

The program of the forest owner is then:

$$\begin{aligned} V_s(\alpha,q) &= pf(R - x(q)R + \alpha x(q)R - (1+\lambda)\alpha p x(q)R - cq) + pg(S-h) \\ &+ (1-p)f(R - (1+\lambda)\alpha p x(q)R - cq) + (1-p)g(S) \end{aligned} \tag{10}$$

The first-order conditions are:

$$\frac{\partial V_s(\alpha,q)}{\partial \alpha} = pf'(B)(x(q)R - (1+\lambda)px(q)R)) - (1-p)f'(A)((1+\lambda)px(q)R) = 0 \tag{11}$$

and

$$\frac{\partial V_s(\alpha,q)}{\partial q} = pf'(B)(-x_qR + \alpha x_qR - (1+\lambda)\alpha p x_qR - c) - (1-p)f'(A)((1+\lambda)\alpha p x_qR + c) = 0 \tag{12}$$

with $B = R - x(q)R + \alpha x(q)R - (1+\lambda)\alpha p x(q)R - cq$, and $A = R - (1+\lambda)\alpha p x(q)R - cq$.

The impact of a variation of c on optimal insurance α_s^* depends on the sign of:

$$\frac{\partial^2 V_s(\alpha^*,q^*)}{\partial \alpha \partial c} = -pf''(B)(x(q)R - (1+\lambda)px(q)R)q + (1-p)f''(A)((1+\lambda)px(q)R)q$$

that is ambiguous.

Using $\frac{\partial V_s}{\partial \alpha} = 0$, we have $(1+\lambda)px(q)R = \frac{pf'(B)(x(q)R-(1+\lambda)px(q)R)}{(1-p)f'(A)}$.
Therefore,

$$\frac{\partial^2 V_s(\alpha^*,q^*)}{\partial \alpha \partial c} = -pf''(B)(x(q)R - (1+\lambda)px(q)R)q + \\ (1-p)f''(A)(\frac{pf'(B)(x(q)R - (1+\lambda)px(q)R)}{(1-p)f'(A)})q$$

and in a more simplified manner we have:

$$\frac{\partial^2 V_s(\alpha^*,q^*)}{\partial \alpha \partial c} = pf'(B)\Big(x(q)R - (1+\lambda)px(q)R\Big)q\Big(- \frac{f''(B)}{f'(B)} + \frac{f''(A)}{f'(A)}\Big)$$

By the definition of the coefficient of absolute risk aversion, $A(x) = -\frac{f''(x)}{f'(x)}$, we have $A(B) = -\frac{f''(B)}{f'(B)}$ and $A(A) = -\frac{f''(A)}{f'(A)}$. Using these definitions we have:

$$\frac{\partial^2 V_s(\alpha^*, q^*)}{\partial \alpha \partial c} = pf'(B)\Big(x(q)R - (1+\lambda)px(q)R\Big)q\Big(A(B) - A(A)\Big)$$

By the first-order condition we know that $x(q)R-(1+\lambda)px(q)R > 0$. We also know that $B \leq A$. Therefore, if preferences exhibit Decreasing Absolute Risk Aversion (DARA), then $A(B) \geq A(A)$. Under this assumption, $\frac{\partial^2 V_s(\alpha^*,q^*)}{\partial \alpha \partial c} \geq 0$.

■

Consequently, under the simplifying assumption of a separable utility for consumption and sentimental value of the forest, insurance and self-insurance are substitutes if the utility function of the forest owner exhibits standard preferences, i.e. DARA. This means that under some assumptions, we obtain the same result as Ehrlich and Becker [23].

3 Conclusion

In this paper, we consider a bivariate utility function with the consumption as the first argument and the forest sentimental value as a second one. We analyse optimal insurance and/or self-insurance decisions in such a framework and we compare our results with the existing results in the literature. We find that some results depend on the cross derivative of the utility function, as ever displayed in health economics (Rey [42]; Eeckhoudt, Rey and Schlesinger [22]; Etner and Spaeter [26]). In particular, we show that under fair premium, full insurance is optimal only if this cross derivative equals zero; otherwise under-insurance and over-insurance may be optimal. Under a positive loading factor, partial insurance is optimal only if the cross derivative is positive; otherwise full insurance may be optimal even with a loading insurance. Then, under some assumptions considering the cross derivative of the utility function, the famous Mossin's theorem is extended to a bivariate utility model. We also observe that risk aversion increases the levels of optimal insurance demand and self-insurance activity, extending this standard result obtained with an univariate utility function to a bivariate utility function. In addition, we analyse the potential substitutability/complementarity between the two risk mangement strategies considered. We prove that when the forest owner can simultaneously insure and invest in self-insurance activity, full insurance is never optimal if the cross derivative is positive. Finally, insurance and self-insurance may be substitutes; and if preferences are separable and exhibit DARA, then insurance and self-insurance are always considered as substitutes.

A potential extension for this study is to consider ambiguity, i.e. uncertainty about the probabilities, rather than to consider risk. Indeed, some evidence indicates that ambiguity has a net impact on insurance behaviour (Cabantous [16]; Cabantous et al. [17]; Hogarth and Kunreuther [28]; Kunreuther et al. [34]). Modelling insurance decisions under ambiguity is an emerging research field with the works of Alary et al. [1] and Brunette et al. [9] which consider an univariate utility function, and with the study of Etner and Spaeter [26] using a bivariate utility function in health economics. However, self-insurance under ambiguity is currently not analysed and then, the link between insurance and self-insurance under ambiguity could be an issue. Another possibility may be to test empirically the substitutability of insurance and self-insurance through experimental economics. Indeed, to our knowledge, such an issue has never been studied.

References

[1] D. Alary, C. Gollier, and N. Treich. The effect of ambiguity aversion on insurance and self-protection. The Economic Journal, 123(573):1188–1202, 2013.

[2] L.H.R. Alvarez and E. Koskela. Does risk aversion accelerate optimal forest rotation under uncertainty? Journal of Forest Economics, 12:171–184, 2006.

[3] G.S. Amacher and R.J. Brazee. Designing forest taxes with budget targets and varying government preferences. Journal of Environmental Economics and Management, 32(3):323–340, 1997.

[4] G.S. Amacher, M. Ollikainen, and E. Koskela. Economics of Forest Resources. Cambridge, MA: MIT Press., 2009.

[5] M. Andersson and P. Gong. Risk preferences, risk perceptions and timber harvest decisions - an empirical study of nonindustrial private forest owners in northern sweden. Forest Policy and Economics, 12:330–339, 2010.

[6] K. Arrow. Uncertainty and the welfare economics of medical care. American Economic Review, 53(5):941–973, 1963.

[7] J. Barreal, M. Loureiro, and J. Picos. On insurance as a tool for securing forest restoration after wildfires. Working Paper, 2013.

[8] T.W. Birch. Private forest-land owners of the northern united states. Bulletin ne-136, Forest Research Service, U.S. Department of Agriculture, Washington, D.C., 1994.

[9] M. Brunette, L. Cabantous, S. Couture, and A. Stenger. The impact of governmental assistance on insurance demand under ambiguity: A theoretical model and an experimental test. Theory and Decision, DOI : 10.1007/s11238-012-9321-8., 2012.

[10] M. Brunette and S. Couture. Assurance et activités de réduction des risques en foresterie : une approche théorique. Revue d'Etudes en Agriculture et Environnement, 86(1):57–78, 2008.

[11] M. Brunette and S. Couture. Public compensation for windstorm damage reduces incentives for risk management investments. Forest Policy and Economics, 10(7-8):491–499, 2008.

[12] M. Brunette and S. Couture. Risk management behaviour of a forest owner to address growth risk. Agricultural and Resource Economics Review, 42(2):349–364, 2013.

[13] M. Brunette, S. Couture, and S. Garcia. La demande d'assurance contre le risque incendie de forêt : Une analyse empirique sur des propriétaires privés en france. Cahier du LEF, 2011-01:18p, 2011.

[14] E. Bryis and H. Schlesinger. Risk aversion and the propensities for self-insurance and self-protection. Southern Economic Journal, 57(2):458–467, 1990.

[15] B.J. Butler and E.C. Leatherberry. National woodland owner survey: 2004 preliminary results. Technical report, Forest Research Service, U.S. Department of Agriculture, Washington, D.C., 2005.

[16] L. Cabantous. Ambiguity aversion in the field of insurance: Insurers' attitude to imprecise and conflicting probability estimates. Theory and Decision, 62(3):219–240, 2007.

[17] L. Cabantous, D. Hilton, H. Kunreuther, and E. Michel-Kerjan. Is imprecise knowledge better than conflicting expertise? evidence from insurers' decisions in the united states. Journal of Risk and Uncertainty, 42(3):211–232, 2011.

[18] H.R. Clarke and W.J. Reed. The trre-cutting problem in a stochastic environment: the case of age-dependent growth. Journal of Economic Dynamics and Controls, 13:569–595, 1989.

[19] P.J. Cook and D.A. Graham. The demand for insurance and protection: The case of irreplaceable commodities. The Quarterly Journal of Economics, 91(1):143–156, 1977.

[20] G. Dionne and L. Eeckhoudt. Self-insurance, self-protection and increased risk aversion. Economics Letters, 17:39–42, 1985.

[21] J.H. Drèze and E. Schokkaert. Arrow's theorem of the deductibe: Moral hazard and stop-loss in health insurance. Journal of Risk and Uncertainty, 47:147–163, 2013.

[22] L. Eeckhoudt, B. Rey, and H. Schlesinger. A good sign for multivariate risk taking. Management Science, 53(1):117–124, 2007.

[23] I. Ehrlich and G.S. Becker. Market insurance, self-insurance, and self-protection. Journal of Political Economy, 80(4):623–648, 1972.

[24] R. Eisner and R. Strotz. Flight insurance and the theory of choice. Journal of Political Economy, 69:355–368, 1961.

[25] J. Englin, P. Boxall, and G. Hauer. An empirical examination of optimal rotations in a multiple use forest in the presence of fire risk. Journal of Agricultural and Resource Economics, 25(1):14–27, 2000.

[26] J. Etner and S. Spaeter. Self-protection and private insurance with ambiguous and non-pecuniary risks. Working Paper, 2012.

[27] P. Gong and K.G. Löfgren. Risk-aversion and the short-run supply of timber. Forest Science, 49(5):647–656, 2003.

[28] R. Hogarth and H. Kunreuther. Risk, ambiguity, and insurance. Journal of Risk and Uncertainty, 2:5–35, 1989.

[29] J. Holecy and M. Hanewinkel. A forest management risk insurance model and its application to coniferous stands in southwest germany. Forest Policy and Economics, 8:161–174, 2006.

[30] R.J. Huang and L.Y. Tzeng. The design of an optimal insurance contract for irreplaceable commodities. Geneva Risk and Insurance, 31:11–21, 2006.

[31] J. Kangas. Incorporating risk attitude into comparison of reforestation alternatives. Scandinavian Journal of Forest Research, 9:297–304, 1994.

[32] E. Koskela and M. Ollikainen. The optimal design of forest taxes with multiple use characteristics of forest stands. Environmental and Resource Economics, 10:41–62, 1997.

[33] E. Koskela and M. Ollikainen. Timber supply, amenity values and biological risk. Journal of Forest Economics, 5(2):285–304, 1999.

[34] H. Kunreuther, J. Meszaros, R. Hogarth, and M. Spranca. Ambiguity and underwriter decision processes. Journal of Economic Behavior and Organization, 26(3):337–352, 1995.

[35] G. Lien, S. Stordal, J.B. Hardaker, and L.J. Asheim. Risk aversion and optimal forest replanting : a stochastic efficiency study. European Journal of Operational Research, 181:1584–1592, 2007.

[36] L. Lönnstedt and J. Svensson. Non-industrial private forest owner's risk preferences. Scandinavian Journal of Forest Research, 15(6):651–660, 2000.

[37] B. Manley and R. Watt. Forestry insurance, risk pooling and risk mitigation options. Technical report, Report prepared for MAF Project CM-09 under MAF POL 0809-11194, 2009.

[38] K. Matsushita, S. Yoshida, M. Imanaga, and H. Ishii. Forest damage by the 13th typhoon in 1993 and forest insurance contracts in kagoshima prefecture [japan]. Bulletin of the Kagoshima University Forests, pages 81–99, 1995.

[39] K.G. Mavromaras. Insurance and protection of irreplaceable commodities: The use of one's own life. Economics Letters, 3:9–13, 1979.

[40] J. Mossin. Aspects of rational insurance purchasing. Journal of Political Economy, 76(4):553–568, 1968.

[41] J. Pratt. Risk aversion in the small and in the large. Econometrica, 32:122–136, 1964.

[42] B. Rey. A note on optimal insurance in the presence of a nonpecuniary background risk. Theory and Decision, 54:73–83, 2003.

[43] M.J. Schelhaas, G.JL. Nabuurs, and A. Schuck. Natural disturbances in the european forests in the 19th and 20th centuries. Global Change Biology, 9(11):1620–1633, 2003.

[44] H. Schlesinger. Optimal insurance for irreplaceable commodities. The Journal of Risk and Insurance, 51(1):131–137, 1984.

[45] F.P. Shioshansi. Insurance for irreplaceable commodities. The Journal of Risk and Insurance, 49(2):309–320, 1982.

[46] R.G. Taylor and J.C. Fortson. Optimum plantation planting density and rotation age based on financial risk and return. Forest Science, 37:886–902, 1991.

[47] Y. Zhang, D. Zhang, and J. Schelhas. Small-scale non-industrial private forest ownership in the united states: Rationale and implications for forest management. Silva Fennica, 39(3):443–454, 2005.