A Unified Theory of Carbon Capture and Adaptation Policies for Climate Change[1]

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Abstract:

We study optimal carbon capture and storage (CCS), taking into account damages incurred from the accumulation of carbon in the atmosphere and exhaustibility of fossil fuel reserves. High carbon concentrations call for full CCS. We identify conditions under which partial or no CCS is optimal. In the absence of CCS the CO2 stock might be inverted U-shaped. With CCS more complicated behavior may arise. It can be optimal to have full capture initially, yielding a decreasing stock, then partial capture while keeping the CO2 stock constant, and a final phase without capture but with an inverted U-shaped CO2 stock. We also introduce the option of adaptation and provide a unified theory regarding the optimal use of CCS and adaptation.

Key words: Climate change, carbon capture and storage, adaptation, non-renewable resources

JEL codes: Q32, Q43, Q54

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1. Introduction

"The Quest CCS Project could be part of the action Alberta and Canada is looking for – to develop valuable oil sands resources with less climate-changing CO2. Quest would capture more than one million tonnes of CO2 per year from Shell's Scotford Upgrader, located near Fort Saskatchewan, Alberta. This is the equivalent to taking 175,000 cars off the road. The CO_2 would be transported safely by pipeline up to 80 kilometers north of the facility to injection wells. It would then be injected more than two kilometres underground where it would be permanently and safely secured under multiple layers of impermeable geological formations". "The Scotford upgrading process adds hydrogen to the heavy oil to break it down into synthetic crude oil which can then be processed into products like gasoline. Producing this hydrogen produces one of the largest sources of CO2 emissions from the upgrader".[4]

Carbon capture and storage (CCS) is generally expected to play a crucial future role in combating climate change. For example, J. Edmonds (Joint Global Change Research Institute) puts forward: "meeting the low carbon stabilization limits that are being explored in preparation for the IPCC 5th Assessment Report are only possible with CCS" (Edmonds, 2008). The main rationale for this view is that the economy is still depending on the use of fossil fuels to a large degree and that it might be too costly to introduce renewables in the short to medium run. CCS would then offer the opportunity to keep on using fossil fuels while limiting the emissions of CO2 into the atmosphere.
CCS consists of several stages. In the first stage the CO2 is captured[5] at point sources, mainly at coal-fired or natural gas-fired power plants, but, as illustrated above, also in the upgrading process of tar oil. For this several technologies are available, including post-combustion capture, pre-combustion capture (oxidizing fossil fuel) and oxy-fuel combustion. In the second phase the CO2 is transported to a reservoir, where in the third phase the captured carbon is stored in for example deep geological formations. A side effect of the latter could be the use of captured carbon for increasing the pressure in oil fields, thereby reducing the cost of future extraction, but at the same time increasing the profitability of enhanced oil extraction, with the subsequent release of carbon, unless captured. As a fourth phase there is monitoring what is going on, once CO2 is in the ground. Each of these phases brings along costs. The economic attractiveness of capture depends on the cost of capture and storage and the climate change damage prevented by mitigation of emissions of carbon. Herzog (2011) and Hamilton et al. (2009) provide estimates of these costs and conclude that the capture cost are about $52 per metric ton avoided (from supercritical pulverized coal power plants), whereas for transportation and storage the costs will be in the range of $5-$15 per metric ton CO2 avoided. This

[4] http://www.shell.ca/home/content/can-en/aboutshell/our_business_tpkg/business_in_canada/upstream/oil_sands/quest/about_quest/

[5] Herzog (2011) points out that already decades ago capture took place, but then the objective was to enhance oil recovery by injecting CO2 in order to increase the pressure in the well.

leads to overall costs amounting to $60-$65 per metric ton. These numbers are more or less confirmed in ZEP (2011). The International Energy Agency (2011) reviews several studies concerned with technologies used on a large scale and finds cost per metric CO2 avoided $55 on average for coal-fired plants and $80 for gas-fired power plants[6]. At the present state of climate change policy CCS is obviously not profitable, but with a carbon price at present of $25 and rising by 4% per year, large scale CCS becomes a serious option before 2040. Nevertheless numerous obstacles remain. Many questions are still unresolved. Some are of a regulatory and legal nature, for example the rights-of-way for pipelines[7], access to the formation where CO2 is injected[8], and how to make the transition from capture megatons in the present to capture gigatons in the future in order to have capture at a level that is substantial enough to combat climate change. Moreover, in Europe the success of CCS also depends of the prevailing CO2 permit price, which at present is low, and has induced Eon and GDF Suez to postpone investments in an EU funded demonstration project near Rotterdam, The Netherlands.
In the present paper we address not so much the development of the CCS technology but the optimal use of the technology once it is available. We only look at capture at point sources, and thereby abstract from geo-engineering, where carbon is captured from the atmosphere. We also assume that a storage technology is available, but cannot be utilized for making fossil fuel reserves accessible at lower cost. Actually, we don't take into account the necessity of (costly) storage capacity that might be limited (see e.g., Lafforgue et al., 2008a and 2008b). We consider both exhaustibility and non-exhaustibility of fossil fuels. BP(2013) estimates that world proved natural gas reserves at the end of 2012, 6,614 trillion cubic feet, are sufficient to meet 56 years of production. Roughly the same holds for oil. For coal the global reserves-to-production ratio is much higher: 109 years. Since climate change is an issue that needs to be addressed in the long term, the assumption of exhaustibility seems warranted, even for coal. However, one could argue that the technically recoverable amounts of gas and coal are much higher and that large part of it will become economically viable due to higher prices or extraction lower costs. For example, the U.S. Energy Information Administration (2013) estimates the technically recoverable amount of gas are huge: 25,000 trillion cubic feet, of which around 30% is shale gas. Given the fact that backstop technologies are becoming cheaper over time, we account for the possibility that not all recoverable resources will be used up, so that from an economic perspective exhaustibility is not taking place.

The criterion for optimality that we use is discounted utilitarianism with instantaneous welfare being the difference between utility from energy use on the one hand and the capture cost and the damage arising from accumulated CO2 in the atmosphere on the other hand. In addressing optimality one

[6] Remarkably, the costs for a project in China are much lower
[7] See N. Jaakkola (2012) for problems that may arise in case of imperfect competition on the transportation network (offshore, in northwestern Europe).
[8] Feenstra et al. (2010) report on the public outcry when plans for storage in the village of Barendrecht (The Netherlands) were revealed.

needs to simultaneously determine optimal capture and storage of CO2. We make a distinction between constant marginal capture cost and increasing marginal capture costs (with marginal capture costs at zero capture zero or positive). Along the optimum a tradeoff has to be made between the direct instantaneous welfare of using fossil fuel on the one hand and the cost of capture and damage caused by the accumulated CO2 on the other. It is found that different assumptions on capture and storage cost lead to considerable differences in the combined optimal capture and storage and extraction regime, in the case of abundant fossil fuel reserves as well as when reserves are limited. We identify cases where in the presence of the CCS it is still optimal to let the CO2 stock increase before partial capture takes place. The core of the paper is section 4 where we derive the optimum for the pivotal case of a finite resource stock and the availability of a CCS technology. There we show that, perhaps surprisingly, it might be optimal to have full capture initially, then partial capture while keeping the CO2 stock constant, and a final phase with no capture but in which the CO2 stock increases initially, before decreasing eventually. Hence the CO2 stock is not inverted U-shaped, as in Tahvonen (1997). In addition to CCS we introduce adaptation as a possible strategy to tackle the climate change problem. Adaptation is concerned with reducing or eliminating climate change induced damages once they occur. Examples of adaptive measures include, the construction of dykes to prevent flooding as a consequence of rising sea levels, and the development of crop varieties that are immune to pathogens that are likely to flourish with changes in certain climate characteristics. It will be shown that this option may lead to postponing CCS, at least CCS at the maximum rate.

The related literature is large. First of all there is the literature that highlights the interrelationship between the use of fossil fuels and climate change (see Plourde (1972), D'Arge and Kogiku (1973), Ulph and Ulph (1994), Withagen (1994), Hoel and Kverndokk (1996), for early contributions). Recently this literature was enriched by explicitly introducing backstop technologies (see e.g., Tsur and Zemel (2003, 2005)) with due attention to the Green Paradox, the problem that may arise if for political economy reasons an optimal carbon tax is infeasible and policy makers rely on a subsidy of the renewable (see e.g. Van der Ploeg and Withagen (2012a and 2012b)). Another step has been set by explicitly incorporating CCS in models with non-renewable natural resources. We start by sketching two recent contributions by Amigues et al. (2012 and 2013), who give a nice up to date survey of the state of affairs and offer a generalization of Chakravorty et al. (2006) and Lafforgue et al. (2008). These papers come close to ours in several respects but at the same time our discussion serves to highlight the essence of our work. Amigues et al. assume that there is a finite stock of fossil fuel, that can be extracted at constant marginal cost. In our case extraction is costless. This is without loss of generality, as the results also hold for constant average extraction costs. They also assume the existence of a backstop technology that is produced at constant marginal cost, which may be high or low. The backstop is perfectly malleable with the extracted fossil fuel and yields utility, together with fossil fuel. We abstract from a backstop technology, but we shall argue that in the case of abundant fossil fuel reserves capture essentially functions as a backstop. Net accumulation of CO2 is the difference between on the one hand emissions, resulting from burning fossil fuel minus the amount captured and stored, and, on the other hand, the natural decay of the stock of CO2, which is a constant fraction of the existing stock. The average cost of capture may take several forms. It may depend just on the amount captured, but, alternatively, one could allow for learning or for scarcity effects. In the former case the average CCS cost is a decreasing function of amount already captured. The latter case captures the fact that with more CCS done in the past it gets more difficult to find new CO2 deposits. We don't allow for stock dependent storage costs, but we do look at different capture cost constellations. Since we concentrate on capture at point sources and not on capture from the atmosphere, net emissions are bound to be non-negative. Apart from the cost aspect, a major difference is in the assumption regarding damages. Amigues et al. put an upper bound, sometimes called a ceiling, on the accumulated CO2 stock, whereas we allow for the stock to take any value in principle, but work with a strictly convex damage function. Conceptually a damage function is more appealing, because it can be constructed in such a way that it includes the ceiling, by taking the damage function almost flat until just before the presupposed ceiling is reached, from where on damage increases steeply. More importantly, Amigues at al. (2012 and 2013) show that for all specifications considered it is optimal not to start with CCS until the threshold is reached. But the main and usual motivation for choosing a ceiling is that it represents a threshold beyond which a catastrophe takes place. Given the many uncertainties surrounding the phenomenon of climate change, this evokes the question whether it is optimal indeed to capture only at the critical level. One of the objectives of the present paper is to investigate this in detail. Our finding is that it might be optimal to do partial CCS at some threshold level, keeping the stock at this level. But after such a phase, the CO2 stock might increase for a while, without CCS taking place.

Other papers addressing CCS include Amigues et al. (2014) and Coulomb and Henriet (2010), who both acknowledge that demand for fossil fuel derives from different sectors of the economy. For example, one sector is the electricity production sector, whereas the other is the transport sector. In the latter capture is far less attractive than in the former. Also in these papers, a ceiling on the CO2 stock is exogenously imposed and capture only takes place at the ceiling in the most likely scenarios. We assume away the existence of a backstop in order to highlight these innovative aspects. Essentially our model is a simple theoretical Integrated Assessment Model of CCS, that also allows for an optimal carbon tax rule representing the social cost of carbon.

The outline of the paper is as follows. We set up the model in section 2. Section 3 deals with the case of an abundant resource, whereas section 4 treats the case of a limited resource. Section 5 deals with the interpretation of CCS as a backstop and shows how adaptation can be incorporated into the model. Section 6 concludes.

2. The model and preliminary results.

In this section we introduce the formal model and provide some first results.

2.1 The model and necessary conditions for optimality.

The social planner maximizes societal welfare, composed of three elements, represented in a separable way. First there is the utility of consuming a commodity produced from a non-renewable natural resource, such as fossil fuel. The second element consists of the cost of capture. Finally, there is the damage from the accumulated stock of pollutants. Social welfare is given by

$$\int_0^\infty e^{-\rho t}\left[u(x(t)) - c(a(t)) - h(Z(t))\right]dt$$

The rate of fossil fuel use is $x(t)$. Extraction cost is zero. Fossil fuel use yields instantaneous utility $u(x(t))$. Capture is denoted by $a(t)$, which brings along a cost $c(a(t))$. The stock of accumulated CO2 is $Z(t)$ causing damage $h(Z(t))$. Damage appears directly in the social welfare function. Alternatively, damage occurs in production (Nordhaus, 2008, and Rezai et al., 2012), but here production is not represented by an explicit production function so that the direct approach is appropriate. Finally, ρ is the constant rate of pure time preference, assumed positive. Regarding the functions involved we make the following assumptions.

Assumption 1.

Instantaneous gross surplus u is strictly increasing, strictly concave and satisfies $\lim_{x\downarrow 0} u'(x) = \infty$ and $\lim_{x\to\infty} u'(x) = 0$.

Assumption 2.

The damage function h is assumed strictly increasing and strictly convex and satisfies $h(0) = 0$, $\lim_{Z\downarrow 0} h'(Z) = 0$ and $\lim_{Z\to\infty} h'(Z) = \infty$.

Assumption 3.

The capture cost function c is strictly increasing and convex.

We allow for different alternative properties within the class defined in this assumption: linear capture costs, as well as strictly convex capture costs (with zero or positive marginal costs at zero capture). The accumulation of CO2 is described by

(1) $\dot{Z}(t) = \zeta x(t) - a(t) - \alpha Z(t), \quad Z(0) = Z_0.$

Here Z_0 is the given initial CO2 stock. The flow of generated CO2 is $\zeta x(t)$. Net emissions are then $\zeta x(t) - a(t)$. Decay of atmospheric CO2 is linear at a constant and positive rate α. This is an heroic assumption[9]. Let $X(t)$ denote the stock of fossil fuel time t and denote the initial stock by X_0. Then

(2) $\dot{X}(t) = -x(t), \quad X(0) = X_0.$

(3) $X(t) \geq 0.$

Since marginal utility goes to infinity as consumption of fossil fuel goes to zero, we don't mention the non-negativity constraint on fossil fuel extraction explicitly. A distinguishing feature of our approach is that we don't allow for capture CO2 from the atmosphere. Hence, only current emissions can be abated. The idea is that CO2 capture at electricity power plants is far less costly than capture CO2 from transportation, for example. So, in addition to non-negativity of capture we impose non-negativity of net emissions.

(4) $a(t) \geq 0.$

(5) $\zeta x(t) - a(t) \geq 0.$

In the sequel we omit the time argument where there is no danger of confusion. The current-value Lagrangian corresponding with maximizing social welfare reads

$$L(Z, X, x, a, \lambda, \mu, \gamma_a, \gamma_{xa}) = u(x) - c(a) - h(Z) - \lambda[\zeta x - a - \alpha Z] + \mu[-x] + \gamma_a a + \gamma_{xa}[\zeta x - a].$$

Here λ is the shadow *cost* of pollution and μ is the shadow *value* of the stock of fossil fuels. The latter vanishes in case of an abundant resource. In addition to equations (1)-(5) we have as necessary conditions

(6) $\frac{\partial L}{\partial x} = 0 : u'(x) = \mu + \zeta(\lambda - \gamma_{xa}).$

(7) $\frac{\partial L}{\partial a} = 0 : \lambda = c'(a) + \gamma_{xa} - \gamma_a.$

(8) $\gamma_a a = 0, \quad \gamma_a \geq 0, \quad a \geq 0.$

(9) $\gamma_{xa}[\zeta x - a] = 0, \quad \gamma_{xa} \geq 0, \quad \zeta x - a \geq 0.$

(10) $\frac{\partial L}{\partial X} = -\dot{\mu} + \rho\mu : \dot{\mu} = \rho\mu.$

(11) $\frac{\partial L}{\partial Z} = \dot{\lambda} - \rho\lambda : \dot{\lambda} = (\rho + \alpha)\lambda - h'(Z).$

(12) $e^{-\rho t}[\lambda(t)Z(t) + \mu(t)X(t)] \quad \rightarrow \quad 0 \quad \text{as} \quad t \rightarrow \infty.$

[9] The process of decay is more complicated in reality, because of all kinds of possible feedbacks and because part of the CO2 stock stays in the atmosphere indefinitely. See Farzin and Tahvonen (1996) for an early economic contribution, basing themselves on Maier-Raimer and Hasselman (1987). For more recent work, see Archer (2005), Archer et al. (2009) and Allen et al. (2009). For a recent discussion of the carbon cycle and its potential consequences for economic policy, see Amigues and Moreaux (2013) and Gerlagh and Liski (2012).

Conditions (6)-(9) are necessary and sufficient to maximize the Hamiltonian with respect to fossil fuel use and capture, yielding that marginal benefits from capture equal marginal cost, if capture is taking place. Equations (10) and (11) are the usual no-arbitrage conditions. Finally, equation (12) is the transversality condition. Note that, taking into account the transversality condition, λ can be written as: $\lambda(t) = e^{(\alpha+\rho)t}\int_t^{\infty} e^{-(\alpha+\rho)s} h'(Z(s))ds$. This is the social cost of carbon, the cost of all future marginal damages due to an increase of emissions at instant of time t .

3. CCS available, abundant resource.

In this section we describe the optimum in the presence of an abundant fossil fuel stock. The results that we obtain are novel, to the best of our knowledge, but still the present analysis mainly serves to highlight in Section 4 the effect of scarcity, which is the main contribution this paper aims to make. Define Z^* by $u'(\frac{\alpha Z^*}{\zeta}) = \frac{\zeta h'(Z^*)}{\alpha+\rho}$ [10]. By virtue of the assumptions made Z^* exists. Actually, Z^* is the (globally stable) steady state of the economy without the CCS option.

INSERT FIGURE 1 ABOUT HERE

Suppose the economy finds itself in Z^* and $\frac{h'(Z^*)}{\alpha+\rho} < c'(0)$. Note that this inequality means that the social cost of carbon, evaluated at the constant Z^*, is smaller than the marginal cost of reducing emissions, so that carbon capture is too expensive and no capture is needed if the economy initially finds itself in $Z_0 = Z^*$ and would stay there or would get smaller over time. It is indeed optimal to take $x(t) = x^* \equiv \frac{\alpha Z^*}{\zeta}, a(t) = 0, \lambda(t) = \lambda^* \equiv \frac{h'(Z^*)}{\alpha+\rho}$ for all $t \geq 0$. All the necessary conditions are satisfied, and they are sufficient as well.

What is optimal for an initial CO2 stock Z_0 smaller than Z^*? Since the CO2 stock is the only state variable, it is monotonic. If the stock would decrease at some instant of time, it would decrease forever. This implies that no capture will take place. Moreover, for a decreasing CO2 stock we would then have $\zeta x(t) < \alpha Z^*$ always, so that the marginal utility of consumption is larger than marginal damages $u'(x(t)) > \frac{\zeta h'(Z^*)}{\alpha+\rho}$ This is clearly suboptimal. Hence, for low initial CO2 stocks it is optimal to let the stock increase up to Z^*.

[10] Appendix A contains a list of critical values of atmospheric CO2.

If the initial CO2 stock is slightly higher than Z^*, it is optimal to have no capture at all. The economy is governed by two equations describing the development of the CO2 stock and the social cost of carbon, equations (1) and (11), where in (11) we have $a = 0$ and x is a function of λ in view of $u'(x) = \zeta\lambda$. We can then determine the highest initial CO2 stock for which this regime is optimal. The path has to end up in the steady state (Z^*, λ^*) and initially the social cost of carbon must equal $c'(0)$. This requires a particular initial value of the CO2 stock, that is denoted by Z^h. If the actual initial CO2 stock is larger than Z^h there will typically prevail a regime with partial capture. In that case we have (1) and (11) again with $\lambda = c'(a)$ and $u'(x) = \zeta\lambda$, giving both a and x as functions of λ. The path has to end in $(Z, \lambda) = (Z^h, c'(0))$ and it must start at $(Z, \lambda) = (Z^m, \lambda^m)$ where λ^m satisfies $u'(x) = u'(a/\zeta) = \zeta\lambda^m$ because at the outset of this period capture is maximal. Finally, for values $Z_0 > Z^m$ we have full capture. Note that with linear capture cost there cannot be partial capture, because that would require a constant social cost of carbon, and thereby a constant CO2 stock.

Next, we look into the case where $\frac{h'(Z^*)}{\alpha + \rho} \geq c'(0)$, so that we may say that capture is cheap. There exist $(\hat{Z}, \hat{\lambda}, \hat{a}, \hat{x})$ with $\hat{a} \geq 0$, such that $\frac{h'(\hat{Z})}{\alpha + \rho} = c'(\hat{a}), u'(\hat{x}) = u'(\frac{\alpha\hat{Z} + \hat{a}}{\zeta}) = \hat{\lambda} = c'(\hat{a}).$ Hence, if the economy finds itself initially in $Z_0 = \hat{Z}$, it is optimal to stay there with partial capture if $\hat{a} > 0$ or no capture if $\hat{a} = 0$. Typically, it is optimal to have partial capture in a region surrounding this steady state if $\hat{a} > 0$. As before, it is possible to determine $Z^m > \hat{Z}$ such that for $Z_0 \leq Z^m$ it is optimal to have partial capture throughout, and for $Z_0 > Z^m$ full capture is required initially. Starting at $Z_0 = Z^m$ we must have $\lambda(0) = \lambda^m$ defined by $u'(x) = u'(a/\zeta) = \zeta c'(a) = \zeta\lambda^m$. Similarly, there exists $Z^a < \hat{Z}$ such that for $Z_0 \leq Z^a$ zero capture prevails, whereas for $\tilde{Z} \geq Z_0 > Z^a$ partial capture is in order. At $Z_0 = Z^a$ the initial social cost of carbon and the initial rate of consumption are determined by $u'(x^a) = \lambda^a = c'(0).$ A typical phase diagram is presented in figure 2.

INSERT FIGURE 2 ABOUT HERE

We end this section with two remarks.

1. With constant marginal CCS cost there is only partial capture in $Z = \hat{Z}$. Hence, in that case $Z^a = Z^m = \hat{Z}$. Also a discontinuity occurs in the capture rate at the steady state stock.
2. With strictly convex CCS cost and $c'(0) = 0$, there is capture throughout, so that $Z^a = 0$.

4. Optimal capture with a finite resource stock

4.1 General approach

Here we consider a finite fossil fuel stock. Tahvonen (1997) studies a world with a finite resource stock as well, but without CCS[11]. Nevertheless his work presents an important benchmark since his assumptions on instantaneous utility and damages are equivalent to ours. For the sake of notation we denote variables of the Tahvonen economy by the superscript T . A first property of the optimum in his model is that, given the initial resource stock, for a low enough initial CO2 stock the shadow price of CO2 (λ^T) is inverted U-shaped over time, whereas otherwise, it is monotonically decreasing. Second, given the initial resource stock, for a low enough initial CO2 stock the CO2 stock is inverted U-shaped over time, and monotonically decreasing otherwise. The intuition is that with a low initial CO2 stock marginal damages from pollution are low compared to the marginal utility of consumption, so that it is in the interest of welfare to consume a lot of fossil fuel initially, at the cost of a higher pollution stock. The possibility of non-monotonicity constitutes a relevant difference with the model with an abundant resource where the shadow price and the atmospheric CO2 are monotonic. In our analysis we need a related result, still in Tahvonen's setting without CCS, namely that, starting with a small initial CO2 stock, $Z_0 < Z^*$, the CO2 stock will initially rise if the resource stock is large enough, and the other way around. The reason is simple. Suppose there exists $Z_0 < Z^*$ such that for all X_0 we have $x^T(0) < \alpha Z_0 / \zeta$, so that in the Tahvonen economy the CO2 stock decreases initially. Then it decreases forever, as demonstrated by Tahvonen, implying from (11) that $\lambda^T(0) \le \frac{h'(Z_0)}{\alpha+\rho}$. It follows from (6) that $\mu^T(0) = u'(x^T(0)) + \zeta\lambda^T(0) > u'(\alpha Z_0 / \zeta) + \frac{\zeta h'(Z_0)}{\alpha+\rho}$. The right hand side of this expression is bounded away from zero, so that the present value shadow price of the resource stock is positive, implying that the initial stock X_0 is finite, a contradiction. Since the extraction rate is continuous over time, it is clear that with a small initial resource stock, it cannot be optimal to have an initial increase of the CO2 stock. Then the claim follows from a continuity argument.

We are now ready to move on to the core of the present paper, which is the full characterization of the optimum in a variety of relevant cases. It turns out to be convenient to define $\tilde{Z}$ by $c'(0) = h'(\tilde{Z})/(\rho+\alpha)$. This will be called the break even CO2 stock, since if atmospheric CO2 would always be above this level, it would pay to capture, whereas if the atmospheric CO2 stock would always be below this level, it would be optimal not to use the CCS technology at all.

[11] Tahvonen (1997) allows for a backstop technology and for stock dependent extraction cost. In describing Tahvonen's contribution we abstract from these issues, because we don't have them in our model.

4.2 Constant marginal capture cost

We first assume that capture is cheap, so that $c = c'(0) = h'(\tilde{Z})/(\rho+\alpha) < h'(Z^*)/(\rho+\alpha)$. The optimum is depicted in figure 3, which gives the optimal trajectories in (Z, X) space.

Several cases are to be considered.

1. Suppose $Z_0 = \tilde{Z}$, the break even CO2 stock. In the absence of CCS and with an abundant resource the CO2 stock monotonically increases from $Z_0 = \tilde{Z}$ to the steady state Z^*, as we have seen in figure 1. With a finite resource stock, we are in the Tahvonen (1997) economy. As explained above, there exists a threshold level, denoted by $\tilde{X}^T$, such that if $Z_0 = \tilde{Z}$ and $X_0 < \tilde{X}^T$ the CO2 stock monotonically decreases along the optimum, whereas if $X_0 > \tilde{X}^T$ the CO2 stock initially increases and after some instant of time it decreases. If $X_0 \le \tilde{X}^T$ then $\lambda^T(0) < c$ because Z^T is decreasing and will therefore always be below $\tilde{Z}$ so that

$$\lambda^T(0) - c = \int_0^\infty e^{-(\rho+\alpha)s}(h'(Z^T(s)) - h'(\tilde{Z}))ds < 0 .$$

Hence, there exists another threshold level $\tilde{X}^{MW} > \tilde{X}^T$, such that $\lambda^T(0) = c$ if $Z_0 = \tilde{Z}$ and $X_0 = \tilde{X}^{MW}$. Stated otherwise, if in the model without CCS the initial resource stock would be $\tilde{X}^{MW}$ and the initial CO2 stock would be $\tilde{Z}$ then discounted damages incurred along the optimum, would equal c. Indeed $\tilde{X}^{MW} > \tilde{X}^T$ because otherwise $Z^T(t) \le \tilde{Z}$ for all $t \ge 0$ and hence $\lambda^T(0) < c.$ We will denote the stable branch in $Z - X$ space, passing through $(\tilde{Z}, \tilde{X}^{MW})$ and leading to $(Z, X) = (0,0)$ by *D*.

INSERT FIGURE 3 ABOUT HERE

Suppose $X_0 = \tilde{X}^{MW}$. It is then optimal to have zero capture forever. The optimum is just the optimum of the Tahvonen economy, starting from the same initial state. So, it is optimal to stay on the curve *D*. By construction $\lambda^T(0) = \lambda(0) = c$ and $\dot{\lambda}(t) = \dot{\lambda}^T(t) < 0$ for all $t > 0$. All the necessary conditions are satisfied and these conditions are sufficient as well.

Suppose next that $X_0 < \tilde{X}^{MW}$. Then it is optimal again to follow the Tahvonen optimal program again, without any use of CCS.

Suppose $X_0 > \tilde{X}^{MW}$. The optimum now consists of two phases. A first phase has partial capture. Along this phase the CO2 stock remains constant at the $\tilde{Z}$ level. The resource stock is reduced until $X(\tilde{T}) = \tilde{X}^{MW}$ at some instant of time $\tilde{T} \ge 0$. The path follows the curve denoted by *E* in figure 3.

After $\tilde{T}$ we are in the Tahvonen economy with the property that the CO2 stock will first increase and then decrease. We should also have $a(\tilde{T})=0.$ Hence, the transition occurs in a smooth way. Along an interval with partial capture, the capture rate and resource use are both monotonically decreasing, as can be seen from (6), with $\lambda-\gamma_{xa}=c$ so that x decreases, and from (1) with Z constant and x decreasing. Hence, the timing of the transition will guarantee continuity.

2. Suppose $Z_0<\tilde{Z}.$ If the initial state of the economy (Z_0,X_0) is to the left of the part of curve *D* that leads to $(\tilde{Z},\tilde{X}^{MW})$, the Tahvonen program is optimal. If the initial state is to the right of that part, it is optimal to have an initial period of time with zero capture, then a period of time with partial capture, moving along *E*, and a final interval of time, starting at $(\tilde{Z},\tilde{X}^{MW})$), with zero capture again, following *D* from the moment of the transition on.

3. To describe the optimal trajectories for the remaining initial stocks we introduce two additional dividing curves, denoted by *F* and *G* in figure 3.

Loosely speaking, the curve *F* is the locus of stocks such that in a regime with full abatement the economy exactly reaches $(\tilde{Z},\tilde{X}^{MW})$. Hence, suppose (Z',X') is on *F*. Consider $\dot{Z}=-\alpha Z$. With $Z_0=Z'>\tilde{Z}$ we can determine the instant of time T' where $Z(T')=\tilde{Z}.$ From T' on, we have zero capture, and full exhaustion of the resource stock. This determines the shadow price $\mu(0)$. Before T' we have $u'(x(t))=\mu(0)e^{\rho t}+\zeta c$, since $\lambda-\gamma_{xa}=c.$ Hence we can uniquely determine X' from

$$\int_0^{T'} x(t)dt = X'-\tilde{X}^{MW} .$$

The curve *G* is the locus of stock values such that for initial stock above *G* there is full CCS, whereas below *G* no CCS is taking place. Obviously, one point of *G* is $(\tilde{Z},\tilde{X}^{MW})$. Moreover, for a very high initial CO2 stock and a small resource stock there will be full CCS. Hence, *G* is decreasing from high Z and low X to $(\tilde{Z},\tilde{X}^{MW})$. It needs to be shown that the decrease is monotonic. This can be seen as follows. Suppose that *G* is non-monotonic. Then there exists Z_1, X_1 and X_2 with $X_2>X_1$ such that (Z_1,X_1) as well as (Z_1,X_2) are on *G*. From $(Z_0,X_0)=(Z_1,X_1)$ as well as from $(Z_0,X_0)=(Z_1,X_2)$ it is optimal to have zero CCS forever. On *G* we have $\lambda=c$. Moreover, with (Z_1,X_1) corresponds a higher $\mu(0)$ than with (Z_1,X_2) because fossil fuel is scarcer. But that implies that on the path originating from (Z_1,X_1) there is less extraction initially, so that the CO2 stock grows less fast and the shadow price λ_1 grows faster. This will reinforce less extraction and lower CO2 accumulation, contradicting that total discounted damages should be the same, since $\lambda_1(0)=\lambda_2(0).$

Thus we may conclude as follows.

Proposition 1.

Suppose constant marginal CCS cost ($c(a) = ca$) and a cheap capture technology Then, when (Z_0, X_0) lies:

-within zone 1, below and to the left of *D* and *G*, it is optimal to never capture CO2.

-within zone 2, between *G* and *F*, it is optimal first to fully capture and next, once a point on the locus *G* is attained, to stop capture forever.

-within zone 3, between *F* and *E*, it is optimal first to fully capture in order to reduce the CO2 stock to the level $\tilde{Z}$, next switch to a partial capture policy, maintaining the CO2 stock at this level until the instant of time where the resource stock has been reduced to $\tilde{X}^{MW}$, from where on capture is no longer necessary.

-within zone 4, between *E* and *D*, no capture is required initially and the CO2 stock increases up to the level $\tilde{Z}$, which, once attained, is maintained for a while thanks to a partial capture policy, capture being given up forever once the resource stock has been reduced to $\tilde{X}^{MW}$.

We now move to the case of an expensive capture technology: $\tilde{Z} > Z^*$. Partial capture is excluded then. The reason is that with partial capture we have $Z = \tilde{Z}, \lambda = \tilde{\lambda} = c$. Hence, with partial CCS we would have $u'(x) = \mu_0 e^{\rho t} + \zeta c = \mu_0 e^{\rho t} + \zeta h'(\tilde{Z}) / (\alpha + \rho) < u'(\alpha \tilde{Z} / \zeta),$ which is incompatible with $\tilde{Z} > Z^*$ since $\zeta h'(Z^*) / (\alpha + \rho) = u'(\alpha Z^* / \zeta)$.

If $Z_0 < \tilde{Z}$ then there is zero capture forever. The reason is that full use of CCS, $a(0) = \zeta x(0)$, would imply $\lambda(0) \geq c.$ Then, the co-state λ is increasing and it will never decrease. Hence there will be full capture forever, which is suboptimal. So, it is optimal now to follow the Tahvonen economy, as is to be expected if the initial stock is below the break-even stock.

If $Z_0 > \tilde{Z}$ and if we would start with zero capture, then CSS will never be used, because the co-state λ monotonically decreases. This occurs if the initial resource stock is small. But, with a large initial resource stock and a large initial CO2 stock, it is optimal to have an initial interval of time with full capture, followed by an interval of zero capture. Hence, another frontier exists between starting with full or zero capture.

Proposition 2.

Suppose constant marginal CCS cost ($c(a) = ca$) and an expensive capture technology. Then there exists a critical level of the CO2 stock, larger than $\tilde{Z}$, and decreasing with the resource endowment X_0, such that:

-for initial CO2 stocks smaller than the critical level there is zero capture forever.

-for initial CO2 stocks larger than the critical level it is optimal to have full capture initially, before switching to a zero capture policy forever.

Remark

If climate change damages are incorporated in the model, not by means of a damage function in the social preferences but though a ceiling: $Z(t) \le \bar{Z}$, then only necessary condition (11) changes. It becomes

(11') $\dot{\lambda} = (\rho + \alpha)\lambda - \pi$

where $\pi(t) \ge 0, \pi(t)[\bar{Z} - Z(t)] = 0$. In the case of constant marginal capture cost capture only takes place at the ceiling. Indeed, suppose that at some instant of time we have $Z(t) < \bar{Z}$ and $a(t) > 0$. Then $\pi(t) = 0$ and $\dot{\lambda}(t) = (\rho + \alpha)\lambda(t)$, implying that λ is increasing. Since $\lambda(t) = c + \gamma_{xa}(t)$ it follows that $\gamma_{xa} > 0$ and increasing, so that there is full capture and the CO2 stock declines. This process goes on, and the threshold will never be reached. Moreover, consumption and capture both go to zero as time goes to infinity, whereas positive consumption, bounded away from zero, is feasible. Hence, there will only be capture at the ceiling. This poses a danger, if the ceiling is motivated by interpreting it as a threshold level, beyond which a catastrophe occurs and if there is uncertainty regarding the effect of capture. More importantly, our model without the ceiling allows for much more complex behaviour of the CO2 stock, as outlined in proposition 1.

4.3 Strictly convex capture cost with $c'(0) = 0$

In this section we consider increasing marginal capture cost, with zero marginal cost at zero capture. Contrary to the case of constant marginal capture cost there will always be some CO2 capture. Actually, it could even be optimal to have full capture indefinitely. The reason is that, because of the limited availability of the resource, the rate of extraction is necessarily becoming smaller over time, so that the effort needed to capture all emitted CO2 gets smaller over time as well, and therefore may be worthwhile. Let us study this possibility in some detail. In case of permanent full capture we have $\zeta x(t) = a(t)$ for all $t \ge 0$. Also $\lambda(t) = c'(a(t)) + \gamma_{xa}(t)$ for all $t \ge 0$ from (7) and (8). Hence, from (6), $u'(x(t)) = \mu_0 e^{\rho t} + \zeta c'(\zeta x(t))$ for all $t \ge 0$. Therefore, the extraction rate x is a function of time and the shadow price μ_0. It is monotonically decreasing over time. The resource constraint $\int_0^\infty x(t)dt = X_0$ uniquely determines μ_0. Consequently, also $x(t)$ and $a(t)$ are determined for all $t \ge 0$ and capture is monotonically decreasing. Moreover, with full capture from the start

$$\lambda(t) = e^{(\rho+\alpha)t} \int_t^\infty e^{-(\rho+\alpha)s} h'(Z_0 e^{-\alpha s}) ds$$

for all $t \geq 0$. In order for permanent full capture to be optimal it must hold that $\gamma_{xa}(t) \geq 0$ for all $t \geq 0$. It is easy to construct an example where this condition is satisfied. Consider the following functions: $c(a) = \frac{1}{2}ca^2, u(x) = x^{1-\eta}/(1-\eta), h(Z) = \frac{1}{2}bZ^2$, where $c > 0, b > 0$ and $\eta \neq 1$ are constants. Then, in the proposed optimum we have $\lambda(t) = bZ_0 e^{-\alpha t}/(\rho + 2\alpha)$. Moreover, $\dot{x}(t)/x(t) \to -\rho/\eta$ as $t \to \infty$ and in the limit $c(a)$ will behave as $e^{-(\rho/\eta)t}$. If Z_0 is large enough and $\alpha < \rho/\eta$ there will always be full capture, because $\lambda = c'(a) + \gamma_{xa} - \gamma_a$ so that $\gamma_{xa}(t) > 0$ for all $t > 0$. Intuitively this makes sense: with a high initial pollution stock, low decay and a large rate of time preference it is optimal to get rid of pollution as soon as possible, and there is not much care for the future. We conclude that it is well possible to have full capture forever. This occurs for high initial pollution stocks and low decay rates. Next the question arises in what circumstances there will always be partial capture. Along any interval of time with partial capture the following holds:

$$u'(x(t)) = \mu_0 e^{\rho t} + \zeta c'(a(t)),$$

$$\lambda(t) = c'(a(t)),$$

$$\dot{\lambda}(t) = (\alpha + \rho)\lambda(t) - h'(Z(t)),$$

$$\dot{Z}(t) = \zeta x(t) - a(t) - \alpha Z(t).$$

For a small initial resource stock the shadow price will be high (μ_0 high). Hence, with a large initial CO2 stock, and therefore a high initial shadow price $\lambda(0)$, it is definitely not optimal to start with partial capture, because it follows from the first two equations that net emissions will be negative then. So, in order to have partial capture throughout the initial pollution stock should not be too large and the resource stock should not be too small. Hence, to conclude, we state

Proposition 3.

Suppose capture costs are strictly convex with $c'(0) = 0$.

There will always be some CO2 capture. Full capture throughout is warranted for high initial CO2 stocks and low decay rates. Partial capture throughout is in order for a low initial CO2 stock and a relatively large resource stock.

4.4 Strictly convex capture cost with $c'(0) > 0$

With increasing marginal capture cost the optimal pattern of capture takes four possible forms, which are summarized in

Proposition 4.

Suppose capture costs are strictly convex with $c'(0) > 0$.

For high enough Z_0 it is optimal to have an initial phase with full capture. Then follows a phase with partial capture, and a final phase with zero capture.
For intermediate levels of Z_0 the optimal sequence is: zero capture, then partial capture and finally zero capture, with the first phase possibly degenerate.
For low levels of Z_0 it is optimal to have zero capture throughout.

Proof

It has been shown before that full capture can only occur at the outset of the planning period. Given that the marginal capture cost at zero capture is bounded away from zero, there should be no capture eventually. From full capture there is no transition possible to zero capture, because that would violate the continuity of the co-state λ. Clearly, for high initial CO2 stocks one should start with full capture. For low initial values of the CO2 stock, we are in the Tahvonen world where capture is not needed. For intermediate initial CO2 stocks it is optimal to build up the stock first, and then to have partial capture. Q.E.D.

5. **Backstops and adaptation**.

CCS helps to prevent or reduce emissions into the atmosphere. We first show that CCS can be interpreted as a backstop technology. Secondly, we demonstrate how the model can be extended so as to incorporate adaptation.

Economically and mathematically, the problem that we have considered thus far is essentially equivalent to the optimal use of a costly backstop technology. If we define $y = x + a/\zeta$ as total consumption, partly originating from the natural resource, x, and partly from a backstop, a, properly scaled, and if the cost of producing the backstop is given by $c(a)$, then we have utility $u(y)$ and accumulation of pollution is given by $\dot{Z} = \zeta(y-a) - \alpha Z$. Hence, a cheaper backstop will always lead to less pollution, as long as the backstop cost is not prohibitively high.

Assume that besides capture it is also possible to adapt to climate damages. This doesn't take place by investing in adaptation capital (see Tsur and Withagen (2013) and Zemel (2014)), but we assume that it occurs through a flow of money outlays $c_y y(t)$, where $y(t)$ is the adaptation effort and c_y is the average cost, assumed constant, and hence equal to marginal cost.
Damages from the accumulation of atmospheric CO2 are given by the function $\hat{h}(Z,y)$. As long as there is no adaptation ($y=0$) this function equals function h of the previous sections. We assume

$$\hat{h}_z(Z,y) = \frac{\partial \hat{h}(Z,y)}{\partial Z} > 0, \hat{h}_{zz}(Z,y) = \frac{\partial^2 \hat{h}(Z,y)}{\partial Z^2} > 0, \text{for all } (Z,y) > 0,$$

$$\hat{h}_y(Z,y) = \frac{\partial \hat{h}(Z,y)}{\partial y} < 0, \hat{h}_{yy}(Z,y) = \frac{\partial^2 \hat{h}(Z,y)}{\partial y^2} > 0, \text{ for all } (Z,y) > 0,$$

$$\hat{h}_{yz}(Z,y) = \hat{h}_{zy}(Z,y) = \frac{\partial^2 \hat{h}(Z,y)}{\partial Z \partial y} < 0, \text{ for all } (Z,y) > 0.$$

Define $\underset{\sim}{h}(Z) \equiv \lim_{y \uparrow \infty} \hat{h}(Z,y)$. Then $\underset{\sim}{h}(Z) > 0, \underset{\sim}{h}'(Z) > 0, \underset{\sim}{h}''(Z) > 0$ for all $Z > 0$.

The social planner's objective is to maximize

$$\int_0^\infty e^{-\rho t}[u(x(t)) - \hat{h}(Z(t), y(t)) - c_a a(t) - c_y y(t)]dt \,,$$

subject to (1)-(5). For the sake of clarity we now denote the constant marginal cost of capture by c_a. Among the necessary conditions for optimality we have the minimization of damage cost and adaptation cost. This is the first step: to derive the optimal adaptation spending as a function of the existing CO2 stock. For a fixed $Z > 0$, a necessary condition for the minimization of $\hat{h}(Z,y) + c_y y$ is

(13) $\quad -\hat{h}_y(Z,y) - c_y \leq 0, \; y \geq 0, \; y(\hat{h}_y(Z,y) + c_y) = 0.$

Hence, if $y > 0$ then $-\hat{h}_y(Z,y) = c_y$. If $-\hat{h}_y(Z,0) < c_y$ then $y = 0.$ So, if the marginal benefit of the reduction of damage at zero adaptation is smaller than its marginal cost, then no adaptation will take place. Figure 4 below depicts the optimal abatement expenditures as a function of the existing CO2 stock $y(Z)$. Let us define the threshold Z_y by $-\hat{h}_y(Z_y,0) = c_y$ and let us assume that $Z_y > 0.$ Then $y(Z) = 0$ for all $0 \leq Z \leq Z_y$ and $y(Z) > 0, y'(Z) > 0$ for all $Z > Z_y.$ Note that $y(Z)$ is continuous at $Z = Z_y$ but not differentiable.

INSERT FIGURE 4 ABOUT HERE

We may also define the net damage function by $\tilde{h}(Z) \equiv \hat{h}(Z, y(Z)) + c_y y(Z).$ We are essentially back in the model analysed in the previous sections, since $\tilde{h}(Z)$ has the same properties as the function $h(Z)$, except possibly for differentiability. As a further illustration let us consider an example. Assume

$$\hat{h}(Z,y) + c_y y = \frac{\vartheta Z^2}{2(y+\varphi)} + c_y y \,.$$

Hence, $Z_y = \varphi\sqrt{\frac{2c_y}{\vartheta}}$ and

$$y = 0,\ \tilde{h}(Z) \equiv \hat{h}(Z, y) + c_y y = \frac{\vartheta Z^2}{2\varphi} \text{ and } \tilde{h}'(Z) = \frac{\vartheta Z}{\varphi} \text{ if } Z < Z_y .$$

$$y > 0,\ \tilde{h}(Z) \equiv \hat{h}(Z, y) + c_y y = Z\sqrt{2\vartheta c_y} + c_y \varphi \text{ and } \tilde{h}'(Z) = \sqrt{2\vartheta c_y} \text{ if } Z > Z_y .$$

Marginal damages are linear initially and then become constant. Let us now make a distinction between two cases

Case a. $c_a > \dfrac{\sqrt{2\vartheta c_y}}{\rho + \alpha}$

Hence, $c_a \geq \tilde{h}'(Z) / (\alpha + \rho)$ for all $Z \geq 0$. In case a CCS cost is higher than the maximal marginal damages from atmospheric CO2. Hence, CCS is expensive relative to adaptation and will never be deployed. The formal argument runs as follows. Suppose $a > 0$ along some interval of time. Then, along that interval, $\lambda = c_a + \gamma_{xa} \geq c_a$. We also have $\dot{\lambda} = (\rho + \alpha)\lambda - \tilde{h}'(Z).$ Moreover, $\tilde{h}'(Z) \leq \sqrt{2\vartheta c_y} < (\rho + \alpha) c_a$ so that $\dot{\lambda} > (\rho + \alpha)(\lambda - c_a) > 0$. Hence λ increases, so that also γ_{xa} increases and there is full capture. This will never come to an end because of the continuity of λ. But this contradicts that eventually we are in the Tahvonen economy with zero capture at low enough pollution stocks. Hence, the optimum is characterized by adaptation prevailing as long as the pollution stock is high, whereas there will be no adaptation if it gets below a certain threshold. CCS is never used, because it is outperformed by adaptation.

Case b. $c_a < \dfrac{\sqrt{2\vartheta c_y}}{\rho + \alpha}$

The solution $\tilde{Z}$ of $c_a = \tilde{h}'(Z)/(\rho + \alpha)$ satisfies $\tilde{Z} < Z_y$. This $\tilde{Z}$ equals the $\tilde{Z}$ defined in the previous section because $\tilde{h}'(Z) = h'(Z)$ for $Z \leq Z_y$. Let us consider several possibilities.

Suppose $Z^* > \tilde{Z}.$ Then CCS is cheap according to the old definition: CCS will be used if the technology would become available in the resource abundant economy's steady state. Moreover, CCS is cheap relative to adaptation. We can reproduce figure 3 and insert Z_y on the vertical axis. This yields figure 5.

INSERT FIGURE 5 ABOUT HERE

The existence of the adaptation option is mainly reflected in the slope of the *G*-curve, the curve along which there was indifference between full and zero capture. Clearly, any path that is optimal in the economy without the adaptation option and that has $Z(t) \leq Z_y$ for all $t \geq 0$ is also optimal in the economy with the adaptation option, because this option is not used. Next, consider optimal paths without adaptation where there is no capture at all, but where the CO2 stock is larger than Z_y at some

instant of time. This holds for example if we would start at the old *G*-curve at a point with $Z_0 > Z_y$.
With the adaptation option in place, it would be used, and, of course, carbon capture will never be optimal. Finally, consider optimal paths in the economy without the adaptation option that will start with full capture and have a CO2 stock is larger than Z_y at some instant of time. This holds for example if we start to the right of the old *G*-curve with $Z_0 > Z_y$. The aim of the economy is to reduce the CO2 stock as quickly as possible, in order to reduce damages. In the new situation there will be adaptation initially This mitigates the damages and therefore also the need to reduce the CO2 stock. Hence, typically, there will be full capture initially, but for a shorter period of time than before. Another way of looking at this is to say that the *G*-curve becomes steeper. To illustrate this, let us fix the initial resource stock and assume that we are in an initial state on the old *G*-curve. Then there is zero capture throughout. In order to have full capture initially, we need a higher initial CO2 stock, i.e. we need to be in a point above the *G*-curve, for the same initial resource stock.
Also the *F*-curve, the path that has full capture and leads to $(\tilde{Z}, \tilde{X}^{MW})$, changes. Note, first of all, that $\tilde{X}^{MW}$ may change itself. Recall that $\tilde{X}^{MW}$ is defined as the initial state from where it is optimal to have zero capture forever and an initial increase of the CO2 stock at the same time, assuming $Z_0 = \tilde{Z}$. We have $\lambda^{MW} = h'(\tilde{Z})/(\alpha + \rho) = c_a$. It could well be that the curve starting in $(\tilde{Z}, \tilde{X}^{MW})$ has $Z(t) > Z_y$ at some instant of time. If so, total discounted marginal damages will be smaller than c_a, so that the new $\tilde{X}^{MW}$ is larger. The shift to the right is then needed to have total discounted marginal damages equal to c_a. But, let us assume for the sake of exposition that $\tilde{X}^{MW}$ is unaffected by the adaptation option. Of course, there is a path with full capture leading to $(\tilde{Z}, \tilde{X}^{MW})$. This is still the *F*-curve in figure 3. However, starting from a point (Z_0, X_0) on the *F*-curve with $Z_0 > Z_y$, it is optimal now to switch to zero capture after $\tilde{X}^{MW}$ is reached, hence to cross the (new) *G*-curve.
Suppose, as a final case, $Z^* < \tilde{Z}$. Here CCS is expensive, but still cheaper than adaptation. Essentially we have the same result as in proposition 2. There will never be partial capture. Zero capture prevails for small enough CO2 stocks and full capture for large enough CO2 stocks. The effect of adaptation is a reduction of the time for which full capture is needed.

Concluding this section, we can say that adaptation can easily be included in our CCS framework. It leads to a different specification of the damage function. In our setting the decisions on CCS and adaptation can be separated in the sense that the adaptation strategy can be decided upon independently of the CCS strategy. The analysis of optimal CCS can then be conducted along the lines of the previous section. We generally find that CCS efforts need to be less strong in the presence of adaptation.

5. Conclusions

In this paper we have given a full account of optimal CCS under alternative assumptions regarding capture cost in the case of an abundant stock of fossil fuels, that cause emissions of CO2. It has been shown that depending on initial conditions and the specification of capture costs optimal policies may differ considerably. In the most realistic case of marginal capture cost bounded far away from zero, no capture is warranted at all. Otherwise, we might have full capture initially, if the initial CO2 stock is high. But eventually capture is partial. If exhaustibility is taken into account, in this case of bounded marginal capture cost, the picture changes. Optimal capture is zero eventually. Hence, any regime with partial capture comes to an end within finite time. With a high initial CO2 stock it is optimal to have full use of CCS. The general picture that arises for the, most realistic, cases where the marginal capture costs is bounded from below, is that the CO2 stock is inverted-U shaped. With a large initial resource stock it will initially increase, CSS is not used, then CCS is used partially, whereas in a final phase no capture will take place. With constant marginal capture cost, the CO2 stock is stabilized at a certain level as long as partial capture takes place, but then definitely the CO2 stock increases for a period of time before approaching zero in the end. Compared with a world where for one reason or another an exogenous upper bound is set for the pollution stock, we find that, if we would put such an upper bound in addition to the damage function, it is well possible to have CCS use before the upper bound is reached.

The implementation of the first-best outcome in a decentralized economy is simple, at least from a theoretical perspective. If the resource extracting sector is competitive and also generates the energy needed by the consumers and owns the CCS technology, then it suffices to impose a carbon tax corresponding with marginal damage, evaluated in the optimum. If the extractive sector, the energy generating sector and the CCS sector are distinct industries, then the same tax needs to be imposed on the energy generating sector.

We have also paid attention to adaptation. It has been shown that adaptation can be represented by modifying the damage function in a straightforward way. Optimal adaptation can be decided upon independently of optimal deployment of CCS. We find that adaptation makes full scale CCS less desirable, in the sense that full CCS is needed for a shorter period of time, at least if adaptation is not prohibitively costly. We have identified a condition for which adaptation is a better option than CCS.

Future research on a large number of issues is in order. A crucial question is where the world's actual initial position is. It should be possible to accurately assess the amount of CO2 that is in the atmosphere at present, as well as the CO2 in the crust of the earth. But, to take the simple case of constant marginal CCS cost amounting to approximately $60, we then still need to specify the global damage function and the estimates of marginal damages vary considerably among studies. Related to this is the fact the CCS and also some types of adaptation require large set up cost, that we haven't taken into account here. Moreover, the model we consider lacks the complexity of the real world. We have treated energy as a commodity that yields utility directly, whereas it should play a role in production rather than in consumption. For the description of the carbon cycle, we have followed an approach that is well established in economics, but, as we have stressed before (see footnote 9), that could be modified according to new insights from climatologists, according to which part of current emissions stay in the atmosphere indefinitely. With an abundant resource this would not lead to outcomes that qualitatively differ from what we found in section 3. We also conjecture that our results go through in case of decay being a strictly increasing and strictly convex function of the existing pollution stock. See Toman and Withagen (2000) on clean technologies and concavity of the self-regeneration function. Another topic for further research concerns the fact that marginal capture cost may be small, but that in order to set up an installation close to the plants that generate CO2, may be costly.

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Appendix A. Definitions of CO2 thresholds.

Z^* solves $u'(\frac{\alpha Z}{\zeta}) = \frac{\zeta h'(Z)}{\alpha + \rho}$.

$\tilde{Z}$ solves $\frac{h'(Z)}{\alpha + \rho} = c'(0)$.

$(\hat{Z}, \hat{a})$ solves $u'(\frac{\alpha Z + a}{\zeta}) = \frac{h'(Z)}{\alpha + \rho} = c'(a)$.

For $Z < Z^a$ capture is zero.

For $Z > Z^m$ there is full capture.

For $Z^a < Z < Z^m$ there is partial capture.

For $Z < Z_y$ there is no adaptation.

For $Z > Z_y$ there is adaptation.

Appendix B. Figures

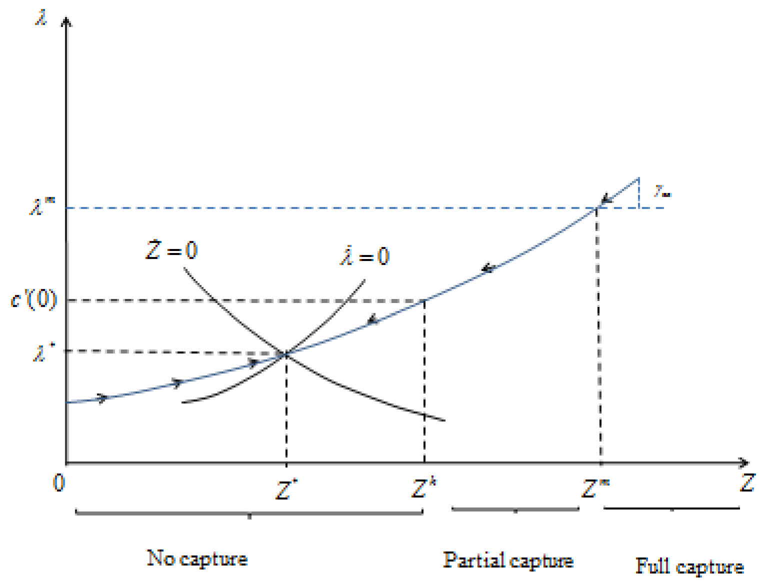

Figure 1: Phase diagram. Case $c'(0) > \lambda^*$, $X_0 = \infty$

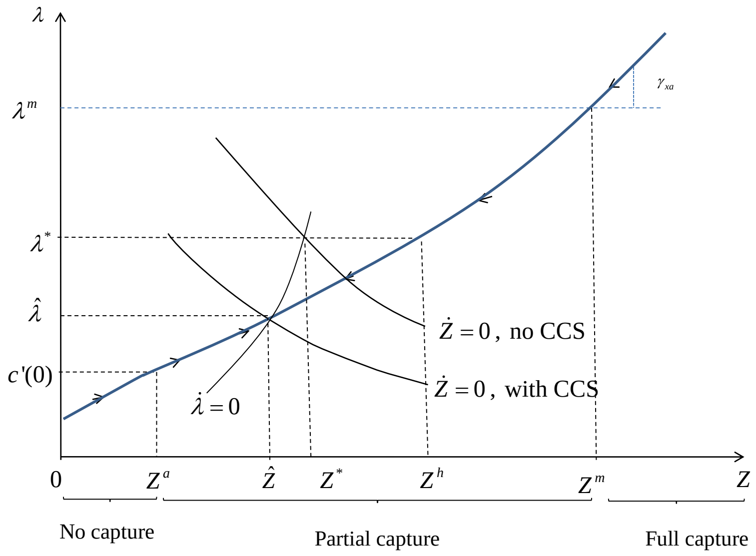

Figure 2: Phase diagram. Case $c'(0) < \lambda^*$, $X_0 = \infty$

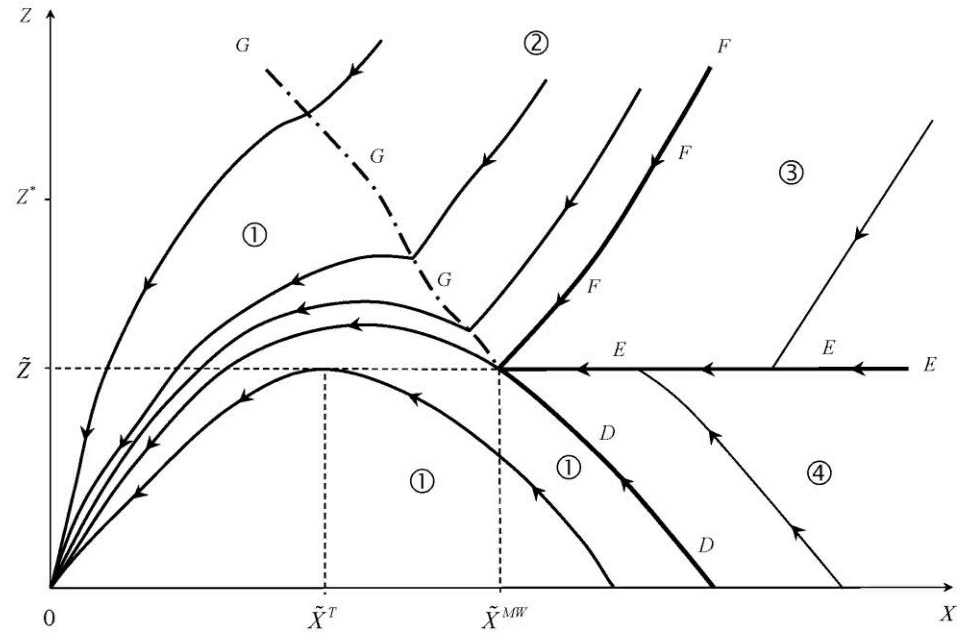

Figure 3: Phase diagram. Finite stock of non renewable resource and low constant marginal capture costs.

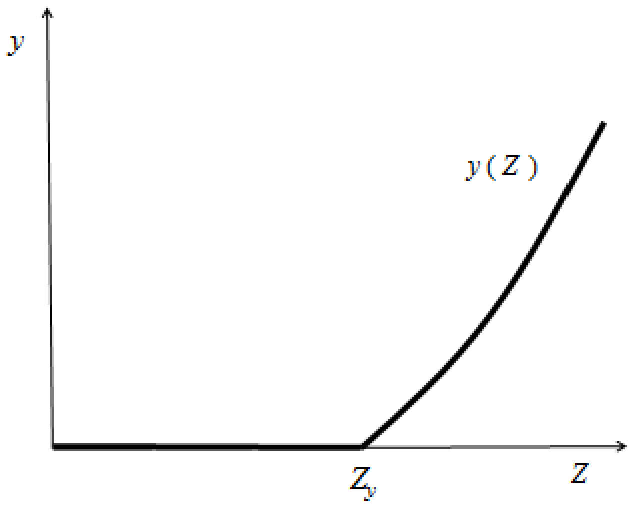

Figure 4. The function $y(Z)$

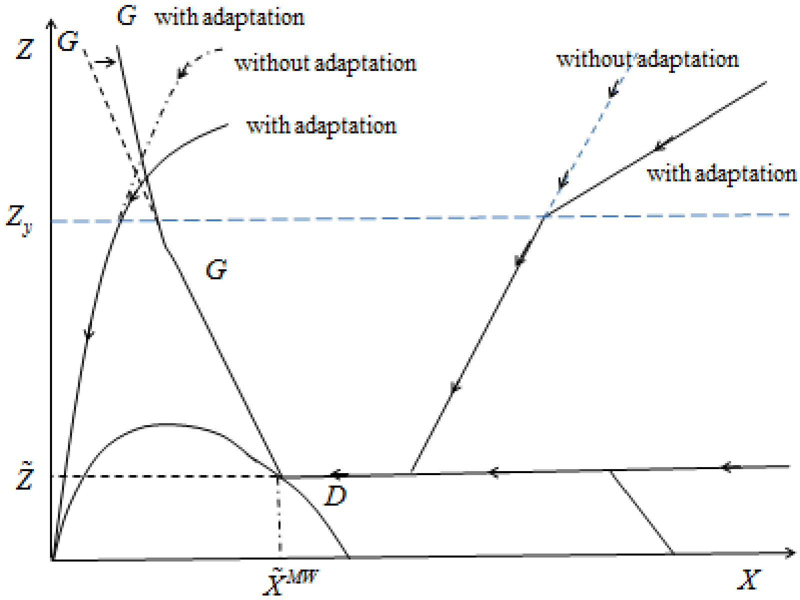

Figure 5. Low constant marginal capture cost, high adaptation cost, $X_0 < \infty$.